Alfred Wegener Institute
for Polar and Marine Research

FEMSECT: a new inverse model to analyze hydrographic section data with velocity measurements based on the finite element method

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1 Overview

Ideally, fluxes through straits or oceanic transects are estimated from direct measurements of velocity. But these measurements are still relatively sparse in the ocean, so that, typically, they have to be interpolated to arrive at transport estimates. On the other hand, transport estimates from high-resolution hydrographic section data and geostrophy suffer from the lack of absolute velocity information. We present an inverse finite element model for single sections that combines both types of data to estimate transports. The model is written in Matlab®; therefore it is highly portable and easily customized to suit the user's needs. After defining a proper cost function and specifying measurement errors and uncertainties associated with the model assumptions, we can use the model to compute not only transport estimates but also their formal errors in an elegant way.

A novel aspect of the model is its use of the finite element method. Among many other advantages, this discretization method allows a flexible computational grid and thereby an accurate representation of the bottom topography, in particular the bottom wedges.

2 Thermal Wind and Finite Elements

Following the general procedure in finite element methods, the thermal wind equations for geostrophic shear are rewritten in weak form

$$\iint \frac{\partial v}{\partial z}\,\tilde{\varphi}\,dx\,dz = -\frac{g}{\rho_0 f}\iint \frac{\partial \rho}{\partial x}(T,S,p)\,\tilde{\varphi}\,dx\,dz, \qquad (1)$$

with the arbitrary test function $\tilde{\varphi}$.

The standard Galerkin method replaces the continuous function v, ρ, and $\tilde{\varphi}$ by an expansion into basis functions ϕ_i that are one at the current node, and zero at all other nodes. For example:

$$v(x,z) = \sum_i v_i \phi_i(x,y) \qquad (2)$$

v_i denotes point values at grid node i. After substitution, Eq. (1) becomes, in matrix notation,

$$\mathsf{U}\mathbf{v} = \mathsf{R}\boldsymbol{\rho}, \text{ or } \mathbf{v} = \mathsf{U}^{-1}\mathsf{R}\boldsymbol{\rho}. \qquad (3)$$

where the matrices U and R are contructed from the basis functions. These matrices act on the coefficient vectors $\mathbf{v}$ and $\boldsymbol{\rho}$. The basis functions ϕ_i are choosen as piece-wise linear functions on all elements or alternatively are chosen to be piece-wise constant on the elements. With the latter choice, our discretization resembles the assumptions of the classical dynamic method.

In order to fit the geostrophic velocity shear to the data, we define the an objective function of the type:

$$\mathcal{J} = \frac{1}{2}(d-m)^T\,\mathsf{W}\,(d-m) + \text{regularization}. \qquad (4)$$

A standard minimization routine finds the minimum of the objective function in the space of the independent control parameter x, which corresponds to the best fit of the model $m(x)$ and the data d, according to the weights W. After the optimization, the formal error can be estimated from the inverse of the Hessian matrix of second derivates of $\mathcal{J}$.

3 Testing the model

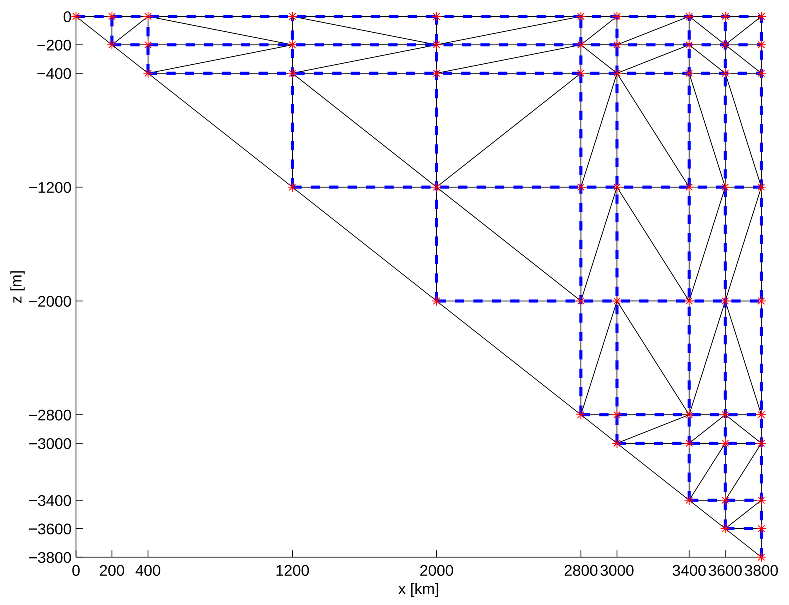

Figure 1: Finite Elements Grid (black lines) and Finite Differences Grid (dashed lines).

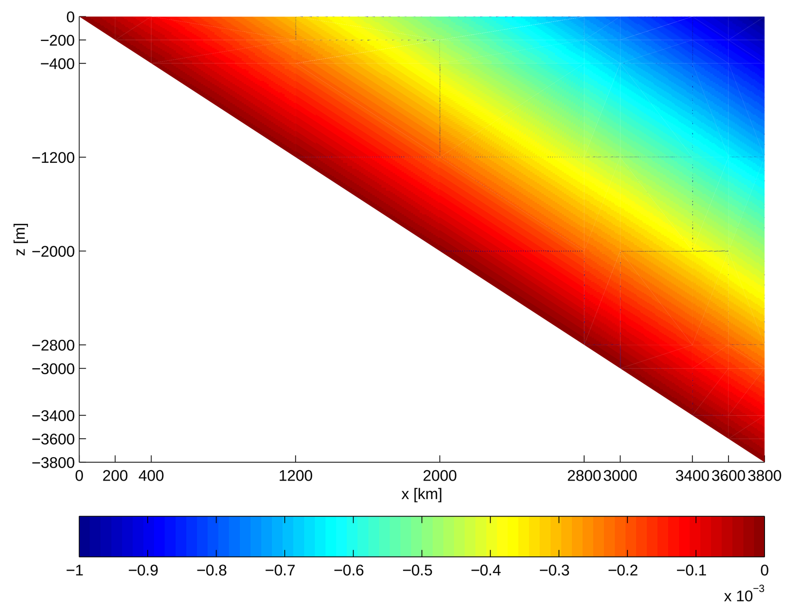

Figure 3: Velocity, obtained by finite element method with piece-wise linear basis functions, [m/s]

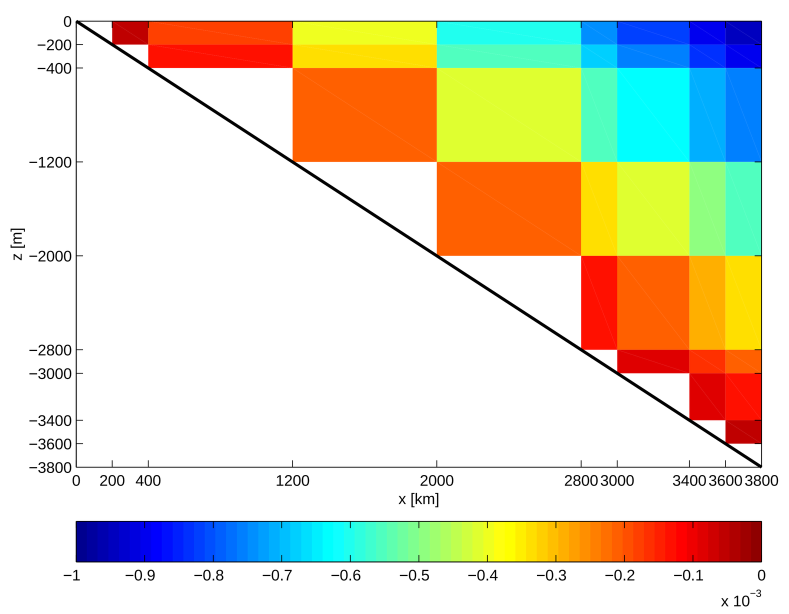

Figure 2: Velocity, obtained by standard dynamics method, [m/s]

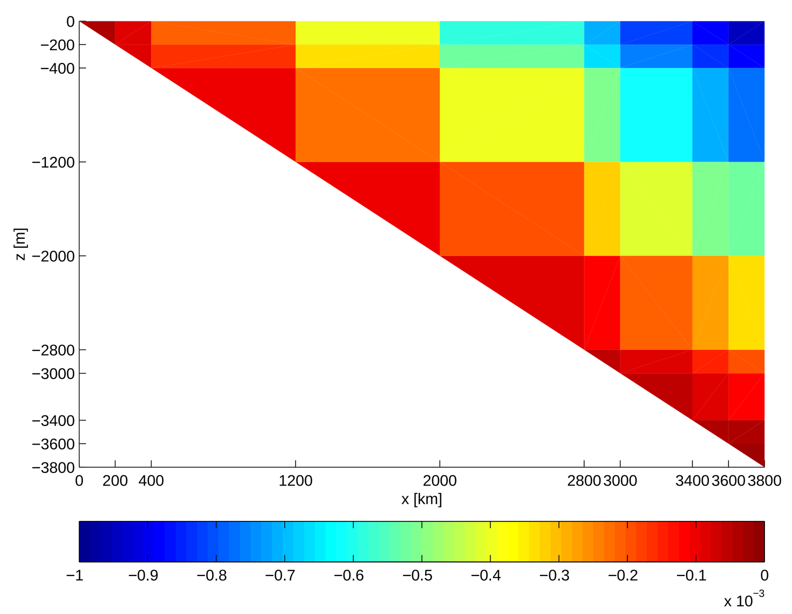

Figure 4: Velocity, obtained by finite element method with piece-wise constant basis functions, [m/s]

The model perfomance was examined using a triangular test domain. It is 3800 km wide and the largest depth is 3800 m. The density field in the section is chosen such that it is linearily growing in the horizontal direction and is independent of depth. The exact solution to the thermal wind equation with no motion at the bottom yields the velocity field with transport of -2.4Sv. For the numerical solution we use a model grid with irregular spacing in both horizontal and vertical directions to imitate the realistic situation of irregularity spaced hydrographic stations (**Fig.1**). Because of the linear density distribution and the perfect representation of the topography by triangles, finite element methods give a perfect result (**Fig.3, 4**).

Difference (model-analytical)

Standart Dynamics method	0.07 Sv,	**Fig.2**
Finite Element method with piece-wise linear velocities	0.00 Sv,	**Fig.3**
Finite Element method with piece-wise constant velocities	0.00 Sv,	**Fig.4**

Remark

Note, that numerical results for a nonlinear density distribution or curved bottom topography will necessarily contain errors which depend on the resolution. Consider the same triangular domain with density given as a quadratic function of horizontal coordinate. The analytical transport with no motion at the bottom is -3.6 Sv. The numerical solutions differ by::

Difference (model-analytical)

Standart Dynamics method	0.10 Sv.
Finite Element method with piece-wise constant velocities	0.03 Sv.
Finite Element method with piece-wise linear velocities	-0.02 Sv.

In both cases the Finite Element methods are superior to the Dynamics Method for geometrical reasons.

4 Transport estimates through Fram Strait

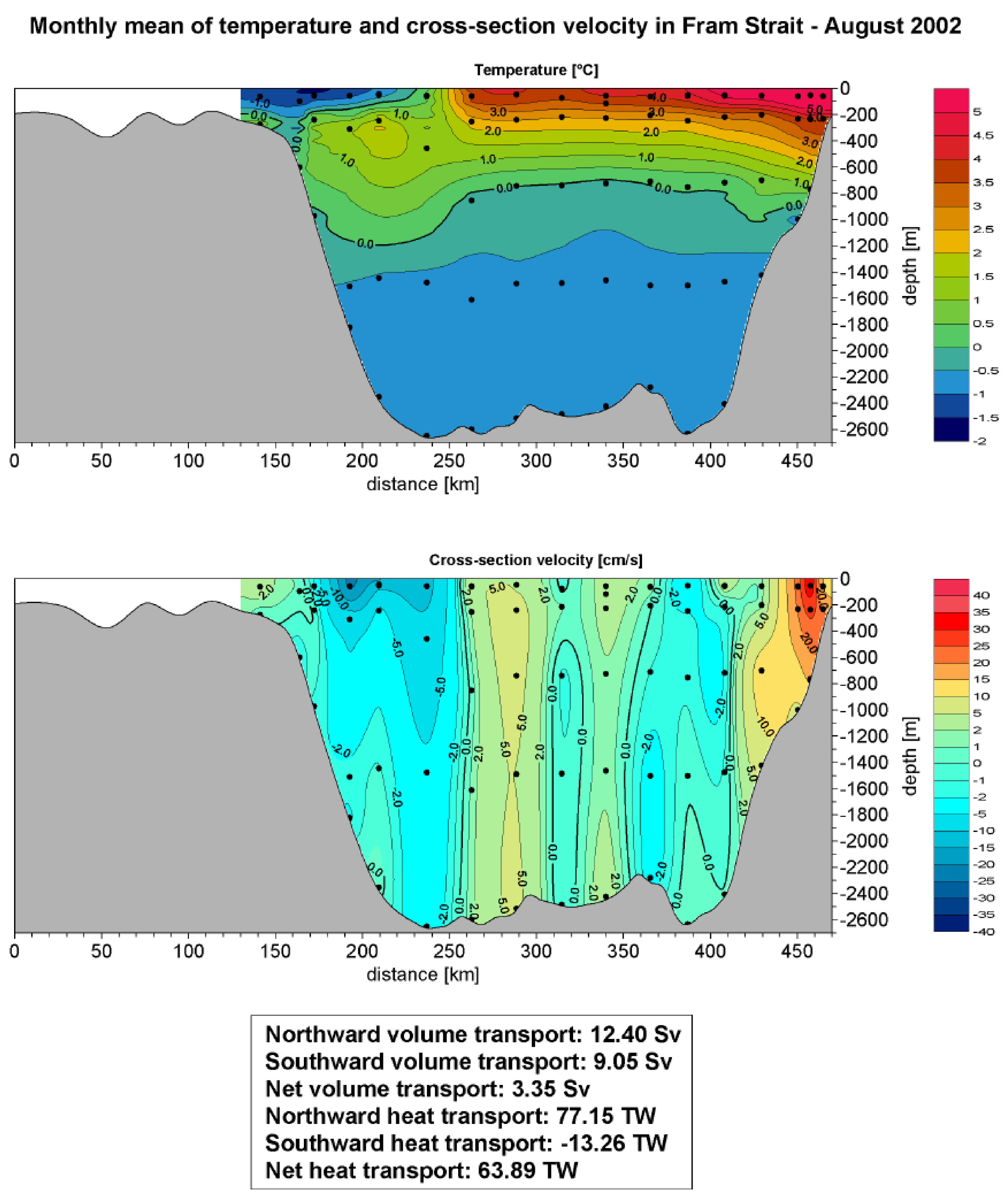

Figure 5: Upper plots - Monthly means of temperature and velocity measured by moored instruments in August 2002. Lower box - Monthly means of nothward, southward and net volume and heat transports in August 2002.

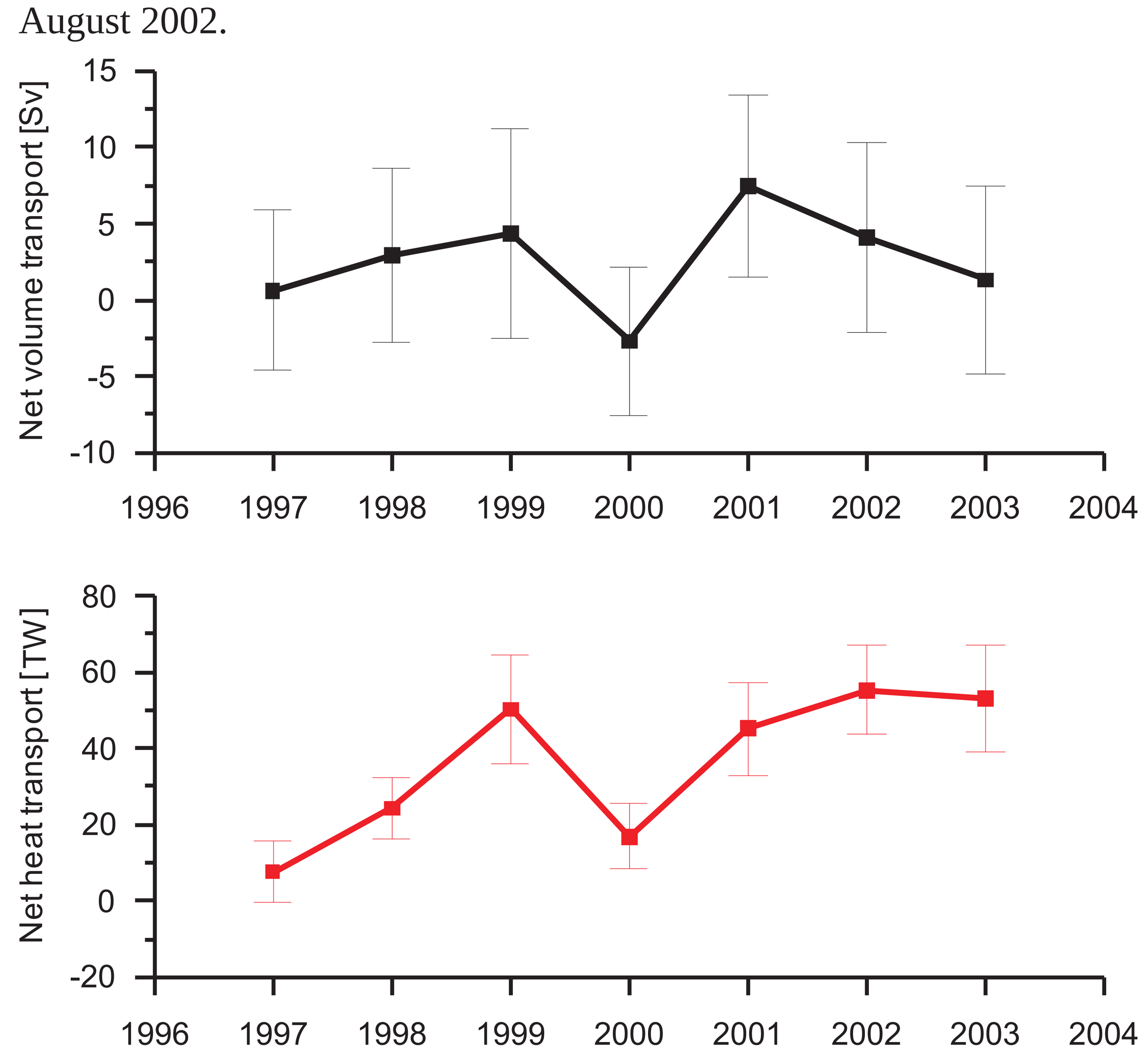

Figure 6: Net volume and heat transports with error estimates, calculated with FEMSECT on a basis of CTD section across Fram Strait (78°50'/79°N) measured in 1997-2003 and monthly averaged velocities from moored current meters. (for relevant periods.)

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Fram Strait is the only deep passage between the Arctic Ocean and Nordic Seas. The warm and salty Atlantic water is carried northward by the West Spitsbergen Current and part of it recirculates immediately within the strait. The cold and fresh polar water is transferred to the south by East Greenland Current. Since 1997 the variability of oceanic fluxes through Fram Strait has been measured by the array of moorings along 78°50'/79°N. Time series of temperature and velocity from moored instruments provide the estimates of heat and volume fluxes with a high resolution in time but the spatial structure of the flow, particularly in the recirculation area, is underesolved. This is a main source of the error in the measured transport. The instantaneous heat and volume fluxes obtained from FEMSECT are based on the high resolution CTD data and referenced to the absolute velocities thus they also include the strong barotropic component dominating in Fram Strait. The FEMSECT results reveal that calculation from mooring data alone tend to overestimate total transports. This is also true for northward and southward transports.

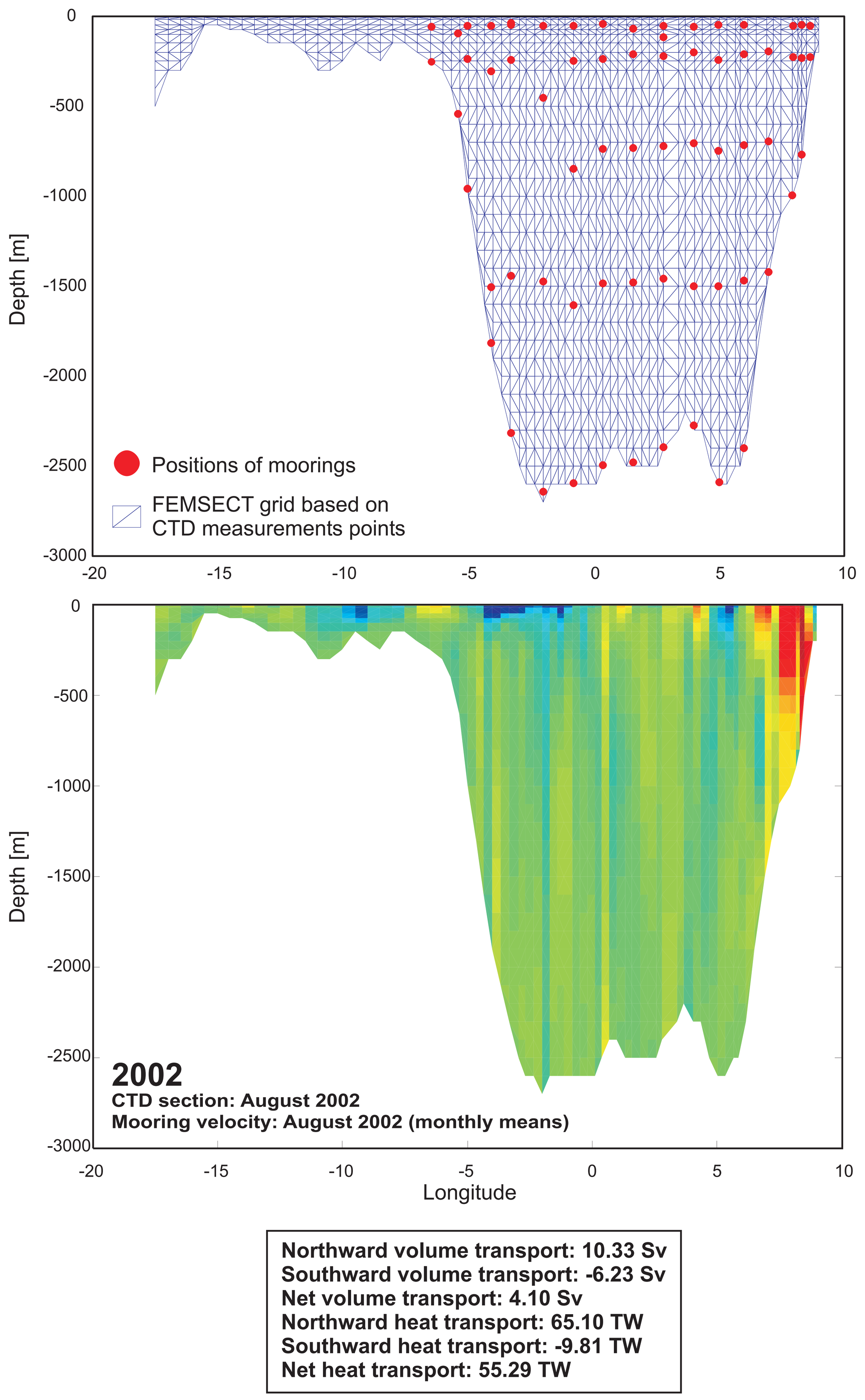

Figure 7: Upper fig. - FEMSECT grid based on positions of CTD measurements in August 2002 and locations of moored instruments in 2002-2003 Lower fig. - velocity field from FEMSECT, inverse solution from temperature and salinity fields measured in August 2002 and monthly means of velocity from moored current meters in August 2002.