FEMSECT: a new inverse model to analyze hydrographic section data with velocity measurements based on the finite element method

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1 Overview

Ideally, fluxes through straits or oceanic transects are estimated from direct measurements of velocity. But these measurements are still relatively sparse in the ocean, so that, typically, they have to be interpolated to arrive at transport estimates. On the other hand, transport estimates from high-resolution hydrographic section data and geostrophy suffer from the lack of absolute velocity information. We present an inverse finite element model for single sections that combines both types of data to estimate transports. The model is written in Matlab, so it is highly portable and easily customized to suit the user’s needs. After defining a proper cost function and specifying measurement errors and uncertainties associated with the model assumptions, we can use the model to compute not only transport estimates but also their formal errors in an elegant way. A novel aspect of the model is its use of the finite element method. Among many other advantages, this discretization resembles the assumptions of the classical dynamic method. A standard minimization routine finds the minimum of the objective function, which corresponds to the best fit of the model on the data, according to the weights W. After the optimization, the formal error can be estimated from the inverse of the Hessian matrix of second derivatives of J.

2 Thermal Wind and Finite Elements

Following the general procedure in finite element methods, the thermal wind equations for geostrophic shear are rewritten in weak form:

\[ \int \frac{\partial v_i}{\partial z} \, dz - \frac{f}{\rho} \int \left( \int \left( \int \frac{\partial v_i}{\partial z} \, dz \right) \, dF \right) \, S \, dz. \]

with the arbitrary test function \( \psi \).

The model performance was examined using a triangular test domain. It is 3800 km wide and 3600 km deep. The density field in the section is chosen such that it is linearly growing in the horizontal direction and is independent of depth. The exact solution to the thermal wind equation with no motion at the bottom yields the velocity field with transport of -2.45 Sv. For the numerical solution we use a model grid with irregular spacing in both horizontal and vertical directions to imitate the realistic situation of irregularly spaced hydrographic stations.


Figure 1: Finite Elements Grid (black lines) and Finite Differences Grid (dashed lines).

Figure 2: Velocity, obtained by standard dynamics method; [m/s]

Figure 3: Velocity, obtained by standard dynamics method; [m/s]

Figure 4: Velocity, obtained by finite element method with piece-wise linear basis functions; [m/s]

Figure 5: Upper plots - Monthly means of temperature and velocity measured by moored instruments in August 2002. Lower box - Monthly means of northward, southward and net volume and heat transports in August 2002.

Figure 6: Net volume and heat transports with error estimates, calculated with FEMSECT on a basis of CTD section across Fram Strait (78°50'79°N) measured in 1997-2005 and monthly averaged velocities from moored current meters. (for relevant periods.)

Figure 7: Upper fig. - FEMSECT grid based on positions of CTD measurements in August 2002 and locations of moored instruments in 2002-2003. Lower fig. - velocity field from FEMSECT, inverse solution from temperature and salinity fields measured in August 2002 and monthly means of velocity from moored current meters in August 2002.

3 Testing the model

The model performance was examined using a triangular test domain. It is 3800 km wide and 3600 km deep. The density field in the section is chosen such that it is linearly growing in the horizontal direction and is independent of depth. The exact solution to the thermal wind equation with no motion at the bottom yields the velocity field with transport of -2.45 Sv.

4 Transport estimates through Fram Strait

Fram Strait is the only deep passage between the Arctic Ocean and Nordic Seas. The warm and salty Atlantic water is carried northward by the West Spitsbergen Current and part of it recirculates immediately within the strait. The cold and fresh polar water is transferred to the south by East Greenland Current. Since 1997 the variability of oceanic fluxes through Fram Strait has been measured by the array of moorings along 78°50'79°N.

Time series of temperature and velocity from moored instruments provide the estimates of heat and volume fluxes with a high resolution in time but the spatial structure of the flow, particularly in the recirculation area, is undersolved. This is a main source of the error in the measured transport. The instantaneous heat and volume fluxes obtained from FEMSECT are based on the high resolution CTD data and referenced to the absolute velocities thus they also include the strong barotropic component dominating in Fram Strait. The FEMSECT results reveal that calculation from mooring data alone tends to overestimate total transports. This is also true for northward and southward transports.