Intercomparison between several finite element and finite volume approaches to model the North Sea tides

Silvia Maßmann, Alexey Androsov, Sergey Danilov and Jens Schröter

Alfred-Wegener-Institut, Bremerhaven, Germany

7th International Workshop on Unstructured Grid Numerical Modelling of Coastal, Shelf and Ocean Flows
Halifax, September 17-19, 2008
Overall goals:

- unstructured, non-linear shallow water models
- adjoint model generation
- optimization of model parameters

What is the focus of this presentation?

- influence of spatial discretization (FE, FV)
- computational efficiency
Unstructured grid models

finite volume (FV): easy to implement, less accurate in space
finite element (FE): implementation more elaborate

FV:
- Chen et al (FVCOM)
- Casulli&Walters (UnTRIM)

FE:
- wave continuity equation models (ADCIRC, QUODDY, MOG2D, T-UGO)
- other models (TELEMAC-2D, P1P1, NC)
finite elements

- $P_1^{nc} P_1$ (NC): approx. 3x more edges than nodes
- $P_1 P_1$: pressure modes, stabilization
finite volumes

UnTRIM
- elevation at circumcenters
- normal velocity at mid edges

FVCOM
- elevation at nodes
- velocity at baricenters
What kind of time stepping provides stable and efficient algorithms?

- semi-implicit: bigger time steps, but matrix inversion (needs a solver)
- explicit: small time steps for stability
- Runge-Kutta: more iterations per time step
- Adam-Bashforth: more storage

<table>
<thead>
<tr>
<th>Leap frog</th>
<th>Runge-Kutta</th>
<th>Adam-Bashforth</th>
<th>semi-implicit</th>
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<tbody>
<tr>
<td>P1P1</td>
<td></td>
<td></td>
<td>x</td>
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<tr>
<td>NC</td>
<td>x</td>
<td>(x)</td>
<td>x</td>
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<tr>
<td>FV</td>
<td>x</td>
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Model intercomparison in the North Sea

M2 tidal wave

- open boundary conditions: TPXO6.2 (OTPS Egbert et al)
- closed boundary condition: free-slip
- bathymetry: GEBCO 1min
Results in the North Sea

Under the limitations

- no wetting and drying
- minimal depth of 5m
- topography not tuned
- constant bottom friction

most of the codes give good results
Error - spatial distribution

Amplitude

NCLF

NCSI

P1P1

FVAB

FVRK

FVSI

TPX
Error - spatial distribution

Phase

- NCLF
- NCSI
- P1P1
- FVAB
- FVRK
- FVSI
- TPX

phase(observation) - phase(computation), deg

Legend:
-20 -105 -90 -75 -60 -45 -30 -15 0 15 30 45
Error statistics

\[
\text{vector error} = \frac{1}{N} \sum_{n=1}^{N} \left[ (A_\ast \cos \varphi_\ast - A \cos \varphi)^2 + (A_\ast \sin \varphi_\ast - A \sin \varphi)^2 \right]^{1/2}
\]
Computational cost

- IBM p655 cluster (5 nodes with 8 CPUs each)
- Use of 1 CPU of a compute node (Power4+ system (1.7GHz) with 16 GByte Ram)
- Size of the mesh
  - Number of nodes = 121699
  - Number of edges = 355589
  - Number of volumes = 233872
Conclusions

- all models give reasonable results
- semi-implicit codes are faster with same accuracy

Outlook

- adjoint model via automatic differentiation
- sensitivity of bottom topography and bottom friction
- optimization of parameters, initial and boundary condition
- wetting & drying (done for NC, under testing)
- astronomical tides (important for the Baltic Sea)
Questions

- How does the energy balance between M2, M4 and M6 look like?
- What is the accuracy of the energy equation?
- What is the influence of numerical diffusion?
- What are good wetting & drying schemes to give correct tidal dissipation rates?