Dependency of tsunami simulations on advection scheme, grid resolution, bottom friction and topography

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Outline

• The tsunami model *TSUNAWI*
  – Numerical concepts and inundation scheme

• The Okushiri tsunami 1993
  – Influence of advection scheme, grid resolution, bottom friction on simulation results

• A worst case scenario for Padang
  – Influence of topography data on inundation

• Conclusion
Shallow water equations

Continuity equation:
\[ \partial_t \eta + \nabla \cdot (\mathbf{v} H) = 0 \]

Momentum equation:
\[ \partial_t \mathbf{v} + (\mathbf{v} \cdot \nabla) \mathbf{v} + f \times \mathbf{v} + g \nabla \eta + \frac{g n^2 |\mathbf{v}|}{H^{4/3}} - \nabla \cdot (A_h \nabla \mathbf{v}) = 0 \]

where
\[ \mathbf{v} = (u(t, x, y), v(t, x, y)) : \text{horizontal velocity} \]
\[ H = h(x, y) + \eta(t, x, y) : \text{total water depth} \]

Boundary Conditions:
\[ \mathbf{v} \cdot \mathbf{n} = \sqrt{\frac{g}{H}} \eta, \quad (x, y) \in \partial \Omega_1 \]
\[ \mathbf{v} \cdot \mathbf{n} = 0, \quad (x, y) \in \partial \Omega_2 \]

Initial Conditions:
\[ \mathbf{v} \big|_{t=0} = 0 \]
\[ \eta \big|_{t=0} = \eta_0 \]
Discretization

Finite element spatial discretization:
non-conforming mixed $P_1$-$P_1^{nc}$ (Hanert et al., 2005)

Explicit time stepping scheme:
Leap frog with Robert-Asselin filter

Inundation: Extrapolation scheme
„Dry node concept“ by Lynett et al., 2002
The Okushiri Tsunami 1993 (Mw 7.8)

- **Field benchmark for the validation of tsunami models** (Synolakis, NOAA, 2007)
  - Initial condition, tide gauge data and bathymetry provided by NOAA
  - Very high runup up to 30m at Monai (west coast of Okushiri island)

Initial uplift distribution

- **max. uplift**: 4.87m
- **max. depression**: -1.12m

Takahashi et al, 1995
Mesh Generation

Mesh refinement is based on the CFL criterion and bathymetry:

$$\Delta x \leq \min \left( k_1 \sqrt{gh}, k_2 \frac{h}{\sqrt{h}} \right)$$

→ fine resolution at the shoreline and at regions of steep bathymetry, coarse mesh in the deep ocean

For the Okushiri testcase, four meshes with different resolution are used:

Mesh 1 (fine_mesh):
- # nodes: 309 410
- min. res. 50m
- max. res. 3km

Mesh 2 (medium_mesh):
- # nodes: 103 361
- min. res. 100m
- max. res. 6km

Mesh 3 (coarse_mesh):
- # nodes: 45 028
- min. res. 150m
- max. res. 9km

Mesh 4:
- # nodes: 214 124
- local refinement in the Monai area:
- min. res. 10m
- max. res. 6km
Fractions of terms in the momentum equation dependent on depth

Locations of different depth on 12 min isochrone
Momentum eq. with and without advection

Divison of nodes into 3 categories:

- depth $< 200$ m
- $200 < \text{depth} < 10$ m
- $10 < \text{depth} < 0$ m

Histograms of mwh: $\eta_{\text{max \ linear}} - \eta_{\text{max \ non-linear}}$

Histograms of max. velocity: \( |v_{\text{max}}|_{\text{linear}} - |v_{\text{max}}|_{\text{non-linear}} \)
Influence of mesh resolution on mwh

\[
\eta_{\text{max}_{\text{fine mesh}}} - \eta_{\text{max}_{\text{medium mesh}}}
\]

\[
\eta_{\text{max}_{\text{medium mesh}}} - \eta_{\text{max}_{\text{coarse mesh}}}
\]

- \( h < 200 \text{ m} \)
- \( 200 \text{ m} < h < 10 \text{ m} \)
- \( 10 \text{ m} < h < 0 \text{ m} \)
Influence of mesh resolution on max. velocity

- $|V_{\text{max}}|_{\text{fine\_mesh}} - |V_{\text{max}}|_{\text{medium\_mesh}}$
- $|V_{\text{max}}|_{\text{medium\_mesh}} - |V_{\text{max}}|_{\text{coarse\_mesh}}$

Legend:
- h < 200m
- 200m < h < 10m
- 10m < h < 0m

Graphs show the percentage distribution of difference in max. velocity [m/s] for different altitude ranges.
Inundation of the Monai area – with and without friction

friction parameter: \( n=0.02 \)

Max. wave height — isolines of topography (0m, 5m, 10m, 15m, 20m)

Runup distribution in the Monai area (in cm)
Inundation of the Monai area – depending on mesh resolution

50 m res. at the coast

Max. wave height
— isolines of topography
(0m, 5m, 10m, 15m, 20m)

Runup distribution in the Monai area (in cm)

10 m res. at the coast

Max. wave height
— isolines of topography
(0m, 5m, 10m, 15m, 20m)
Worst case tsunami scenario for Padang, Sumatra

\[M_w 8.98\]

Max. Uplift = 3.73 m
Max. Depression = -1.60 m

Variable resolution of the mesh:
~57 m in Padang region
~7 km in deep sea
Worst case tsunami scenario for Padang

Topography and inundation results

SRTM (90 m res.)

HRSC (50 m res.)
Conclusion

- Advection is important in shallow water
- Grid resolution has effect on mwh and velocity in coastal regions
- To simulate runup successfully, a fine mesh resolution is needed
- Good topography data is crucial for reliable inundation results