Numerical simulation of double-diffusive Processes in Ocean and in Stars
Metstrm Meeting Berlin

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Aim of this work / open questions

- Understanding mixing processes in various double-diffusive regimes (star, ocean, coffeecup).
- What is the role of relevant dimensionless numbers according to mixing time-scales?
- How efficient is double-diffusive convection? (power laws, $\kappa_{\text{eff}}$)
- Is double-diffusive convection in liquid and gaseous regimes comparable?
Saltfingers and Semiconvection

- **Saltfingers** occur when the **faster** diffusing component stabilizes and the **slower** diffusing component destabilizes.
- **Semiconvection** occurs when the **slower** diffusing component stabilizes and the **faster** diffusing component destabilizes.

**Figure:** Saltfinger instability
Characteristic of Saltfingers

Staircases

- Internal instabilities by growing Saltfingers
- Saltfingers producing staircase structures in the ocean, probably
- Interface in staircase is an area of vertical salinity transport by Saltfinger (resp. Semiconvection)
Effective Diffusivities

**Effective Diffusivity $K_T$**

- SF-layer
- SF-linear
- SF-3
- SF-4
- SF-5

**Effective Diffusivity $K_S$**

- SF-layer
- SF-linear
- SF-3
- SF-4
- SF-5

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Turbulent mixing by Saltfingers

Arising Saltfingers (left), turbulent mixing (middle) and equilibrium state (right)
Results for Saltfingers and Semiconvection

• possible parametrisation for effective diffusivities

\[ K_S = f(R^f_\rho, \gamma, \tau), \quad \text{and} \quad K_T = \frac{\gamma}{R^f_\rho} K_S \]

• estimated effective diffusivities are comparable with measurements and labor experiments of Saltfingers

• Semiconvection leads to stable stratification (see below)
**Semiconvection**

**Properties:**

- 'stabilized convection' - counteracting concentration gradient
- Semiconvection leads to layering (Latte Machiatto)
- $\kappa_S \ll \kappa_T$
- $R_\rho = \frac{Ra_S}{Ra_T} > 1$
- $\sigma$ and $\tau$ [10$^{-2}$ – 10$^1$]

*Figure:* Semi-convective double layer simulation. (Salinity/Temperature)
How do $\sigma$, $\tau$, $R_\rho$ and $Ra_T$ influence the SC mixing process?

To answer this question 2D numerical simulations have been done. The influence of the initial parameter space is measured in terms of the Nusselt numbers $Nu_T$ and $Nu_S$.

- non-dimensional approach for idealized water and gaseous regimes
- compressible / incompressible (Boussinesq approximation / fully explicit ideal gas)
- wide range of $\sigma$, $\tau$, $R_\rho$ and $Ra_T$
- Question 1: $Nu_T \sim Ra_T$ (power law)?
- Question 2: $Nu_T \sim Nu_S$?
Mathematical approach (Antares code)

- Low Mach number solver for binary mixture equations (explicit and implicit)
- Gaseous SC firstly solved in characteristics with WENO 5th order.
- Time integration: TVD2, optional: Boussinesq equations solved with SDIRK (fixed point iteration)
- Numerically stable solutions on staggered grid for WENO and Poisson solver. (BiGrid Marker And Cell)
- Parallelisation: Efficient Poisson solver based on Schur Complement method. Hybrid parallelisation (OpenMP and MPI).
- Local grid refinement
Compressible and incompressible SC regimes are (numerically and physically) comparable as long as $H < H_P$.

$N u_S = \tau^{-1/2}(N u_T - 1)$

Stable multilayer simulations for $\sigma < 1$.

An extrapolation into stellar relevant regime is valid (under the diffusion correction assumption) and has already been done (next slide).

Next step: Direct layer formation, 3D.
Modified power law: $\text{Nu}_T = (\sigma R_a T)^{-0.25} + 1$
for $R_a = \sigma R_a T < 10^6$ in the limit $R_\rho \downarrow 1$
Comparison between Saltfingers and Semiconvection

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Publications and Presentations

Theses:
T. Zweigle: Direkte Numerische Simulation von Salzfingern im Ozean
F. Zaussinger: Numerical simulation of double-diffusive convection (submitted)

Publications:
F. Zaussinger, H. Spruit: The mixing rate of Semiconvection (to be submitted to Astronomy & Astrophysics in Nov/Dec)
Cooperations

- Egbers / Harlander (BTU Cottbus)
- Behrens / Wirth and Horenko / Klein / Munz: Model intercomparison study. Test case: Bubble test case from Robert (1992)

Figure: Saline bubble experiment for increasing resolution ($62^2 \rightarrow 1000^2$)