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An Algorithm for Approximate Paleotemperature Calculations of Mountain Rock from Temperature Measurements in Deep Bore Holes.

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Abstract: The derivation of palaeotemperatures from borehole temperature inversion is ambiguous in principle. An algorithm is presented which allows to reduce ambiguity by making use of a number of borehole temperature profiles from the same area.

Zusammenfassung: Die Ableitung von Palaeotemperaturen aus Temperaturprofilen in Bohrlöchern gelingt wegen der Mehrdeutigkeit inverser Lösungen nur unter gewissen Annahmen. Hier wird ein Algorithmus vorgestellt, der es unter der Verwendung von Temperaturprofilen mehrerer Bohrlöcher erlaubt, die Mehrdeutigkeit einzuschränken.

INTRODUCTION

There are several physical and mathematical models of heat exchange in rocks that lead to approximate solutions for temperature values in past geological epochs based on present-day temperature profiles.

To solve the associated inverse heat conductivity problem (INGERSOLL et al. 1954), a one-dimensional model of heat exchange is used without taking into account the water transition phase to calculate the postglacial time period. The approximate paleotemperature formula is constructed by using the solution to Stephan's problem with simplifying assumptions (BALOBAEV 1985).

In inverse theory, there are difficulties of existence and unambiguous solution. At the same time, in heat exchange process modelling for a considerable time interval T, as a rule, there is no information about the change in the character of many parameters of the system. These parameters are the rock-water physical characteristics: density, moisture, heat conductivity l, thermal heat capacity C. Not only is there an absence of reliable data, but the many system parameters have emergency features (KAPUSTINA et al. 1989).

The last position makes more complicated the explanation and estimation of data reliability from the modelling effort. Thus, the general solution to the restoration problem based on present-day temperature profiles of paleotemperatures rests on the use of determined models of heat conductivity process in rocks, i.e. with a system approach to the problem.

DISCUSSION OF MODELS

The heat-exchange process model is expressed by the corresponding boundary value problem of the heat-conductivity equation. In this case, there should be a mixed boundary value problem corresponding to conditions when an unknown temperature T (t) is given on the surface l_0 and at a depth l_1 there is unknown geothermal gradient g (t).

Consider the process within the limits of its single-measured model of the only conductive heat-exchange without water phase conversion. The heat exchange equation following from this assumption will contain l and C and will not depend on the temperature t (x, t) at a depth of x, $l_0 \leq x \leq l_1$, assuming that the surroundings are without sources and heat flows.

In connection with the necessity of quantitative determination of post pleistocene climate changes, beginning with the time of maximum development of Würm glaciation (20,000 B.C.), mathematical modelling does not take into account the seasonal climate changes so the field of research is stratum from a depth of l_0 (zero annual amplitudes), $l_0 \approx 10$ m, to a depth of $l_1 < 400$ m, where temperature changes (period about 5400 years) penetrate, provided the coefficient of temperature conductivity is 0.002 m^2/h. This follows from the formal application of the Fourier law.

Additional assumptions are necessary. The group average of unknown paleotemperature on the soil surface (i.e. the average value has similarly changed during some intervals) is supposed to be equal to the temperature at a depth of „zero" annual amplitudes. Just as we assume that the geothermal gradient is a piecewise-continuous time function, but „l" and „C" depend only on a depth of x. Thus, a one-dimensional model of heat-exchange process close to an investigated one is considered (INGERSOLL et al. 1954).

METHODOLOGY

Let N be the given quantity of unknown time intervals $z_j - z_{j-1}$, during which the temperature on the soil surface was similar. T_j are the unknown group averages and G_j are the unknown averages of geothermal gradient concerning the same time intervals ($j = 1, 2, ..., N$, $z_0 = 0$). Let the field limits l_0 and l_1 be given along with N, the net versus $w = \{l_0 = x_1, x_2, ...x_k = l_1\}$ in that the soil temperature is known in the modern period as a discrete function $t^* (x_m, z_N)$, $m = 1, 2, ..., K$, where „K" is the number of temperature data; besides physical characteristics $\lambda(x)$, $\lambda(x) > \lambda_0 = const > 0$ and $C(x)$, $C(x) > C_0 = const > 0$, $l_0 \leq$

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$x \leq l_1$ are known. Then it is necessary to find z_j, T_j, G_j, j = 1, 2, ..., N under condition of the minimum quadratic mean functional $F(z_j, T_j, G_j)$:

$$F(z_j, T_j, G_j) = \sum_{m=1}^{K} [t(x_m, z_N) - t^*(x_m, z_N)]^2 \qquad [1]$$

j = 1, 2, ..., N, where $t(x_m, Z_N)$ is function $Z_1, T_1, G_1, Z_2, G_2, ..., Z_N, T_N, G_N$, which is determined from the mixed boundary value problem for the heat-conductivity equation. Let us formulate this problem. Let parameters of the limiting conditions Z_j, T_j and G_j be known, then it is required to find t(x, t) i.e. the soil temperature, satisfying the conditions of the following mixed boundary value problem:

$$c(x)\frac{\partial t}{\partial t} = \frac{\partial}{\partial x}[\lambda(x)\frac{\partial t}{\partial x}],\ l_0 < x < l_1,\ 0 < \tau \leq Z_N \qquad [2]$$

$$t(x,0) = T_1;\ \tau(l_0, \tau) = T_j;\ Z_{j-1} < \tau \leq Z_j;\ j = 1, 2, ..., N; \qquad [3]$$

$$\frac{\partial t}{\partial x}_{x-l1} = G;\ Z_{j-1} < \tau \leq Z_j;\ j = 1, 2, ..., N. \qquad [4]$$

The minimum of the special purpose function [1] at $Z_0 < Z_1 < ... < Z_N$ may be found by method of in-coordinate descent, which does not require a derivative calculation. First, this method requires putting the initial values of the unknown point of minimum Z^0_j, T^0_j, G^0_j. Then, the consecutive single-valued minimization of the function on each coordinate is fulfilled. This process results in initial values Z^1_j, T^1_j, G^1_j, for the following iteration. If the values of parameters Z_j, T_j and G_j for two consecutive iterations are close, then the process is completed.

When calculating i^{th}-iteration for [1], it is necessary to find the numerical solution of equations [2] - [4]. For a solution a differential scheme for [2] - [4] is drawn up with the first order of time precision and the second order of depth precision. The resultant system of linear algebraic equations for the discrete values of net temperature is derived by the driving away method. In differential scheme, as a rule, margin condition of the first order [3] is precisely approximated. The boundary condition of the second order (Samarsky & Gulin 1989) is roughly approximated with the second order of precision in depth and with the first order precision in time. This approximation prepared for the driving away method is given by the following expression:

$$t^j_k = \frac{1}{1 + \frac{h^2}{2ta^2}} t^j_{k-1} + \frac{G_j h + \frac{h^2}{2ta^2} t^{j-1}_k}{1 + \frac{h^2}{2ta^2}} \qquad [5]$$

where,

$$t^j_k = t(x_k, Z_j);\ \tau = Z_j - Z_{j-1};\ h = x_k - x_{k-1};\ a^2 = \frac{l(x_k)}{c(x_k)}$$

The temperature versus depth profile usually is collected from a limited number of holes at an unequal distance from one another and a detailed temperature table is drawn up with the aid of Lagrange interpolation of the third degree.

RESULTS AND CONCLUSIONS

For the solution of this algorithm, the mixed program ALGOL-60 and FORTRAN-IV was developed for running on a BESM-6 computer under the „Dubna" monitor. In this program, the algol module is used for size-less input data; in this way, data treatment of three temperature profiles is provided. FORTRAN modules provoke modules of vectorial algebra. Included in MNOGR-module in the fifth edition of Program Library a FORTRAN language in the Scientific Research Cybernetics Center of Moscow State University. This module expresses algorithm of minimization of function of many variables without derivatives counting at two-sided limits on variables by the method of coordinate descent. The program provides graphic data presentation with the aid of a plotter.

The key feature of the program is the transformation of two-measured massives in one-measured ones and vice versa its correct transmission to subprograms with formal massives having controlling size. For this purpose, the call of library subprogram ARRAY is used.

The program assumes marginal temperature treatment, determined not more than by the eleven averages. For differentiation with respect to depth 91 modes are used, with respect to time 41. Calculation for one variable at each mode takes 2 minutes of CPU time.

The program of global climate change research is an original method of investigation. This program permits one to analyze a great number of geothermal sections and present the temperature dynamics on the Earth's surface over a long period of time.

From the beginning of the research, paleotemperature restoration was carried out in several sites of the Arctic. According to Baulin et al. (1967), Nekrasov & Deviatkin (1974), Taylor & Judge (1974) some wells were chosen near the Ob mouth (Salehard, Russia), the Lena mouth (Tixi, Russia), the lower Mackenzie (Inuvik, Reindeer 1, Canada), Melville (Winter Harbour, Canada).

References

Balobaev, V.T., (1985): About temperature restoration of frozen rocks. „Cryolithozone Development in Upper Cenozoic." M. Science,

Baulin, V.V., Belopukhova, E.B., Dubikov, G.I. & Shmelev, L.M. (1967): Geocryological conditions of Western-Siberian plain.- 214 pp., Nauka, Moscow.

Budico, M.Y. (1980): The Climate in the Past and in the Future. L., Hydrometeoizdat.

Ingersoll, L.R., Zobel, O.J. & Ingersaoll, A.C. (1954): Heat conduction with engineering, geological and other applications.- Madison, the University of Wisconsin Press.

Kapustina, N.I. et al. (1989): The subtlety of system analysis and paradox of the average „The system approach to geology." M. Science.

Nekrasov, I.A.& Deviatkin, V.N. (1974): Morphology of the Cryolithzone in the Yana River basin and adjacent regions.- 72 pp., Nauka, Novosibirsk.

Samarsky, A.A. & Gulin, A.V. (1989): Numeral methods. M. Science.

Taylor, A.E. & Judge, A.S. (1974): Canadian geothermal data collection-Northern Wells, 1955 to February 1974.- Geothermal Series, 1, 171 pp., Dept. of Energy, Mines and Resources, Ottawa, Canada.

Turchak, L.I. (1987): Principles of numeral methods.- V.V.M. TCHENNIKOV (ed.), Science.