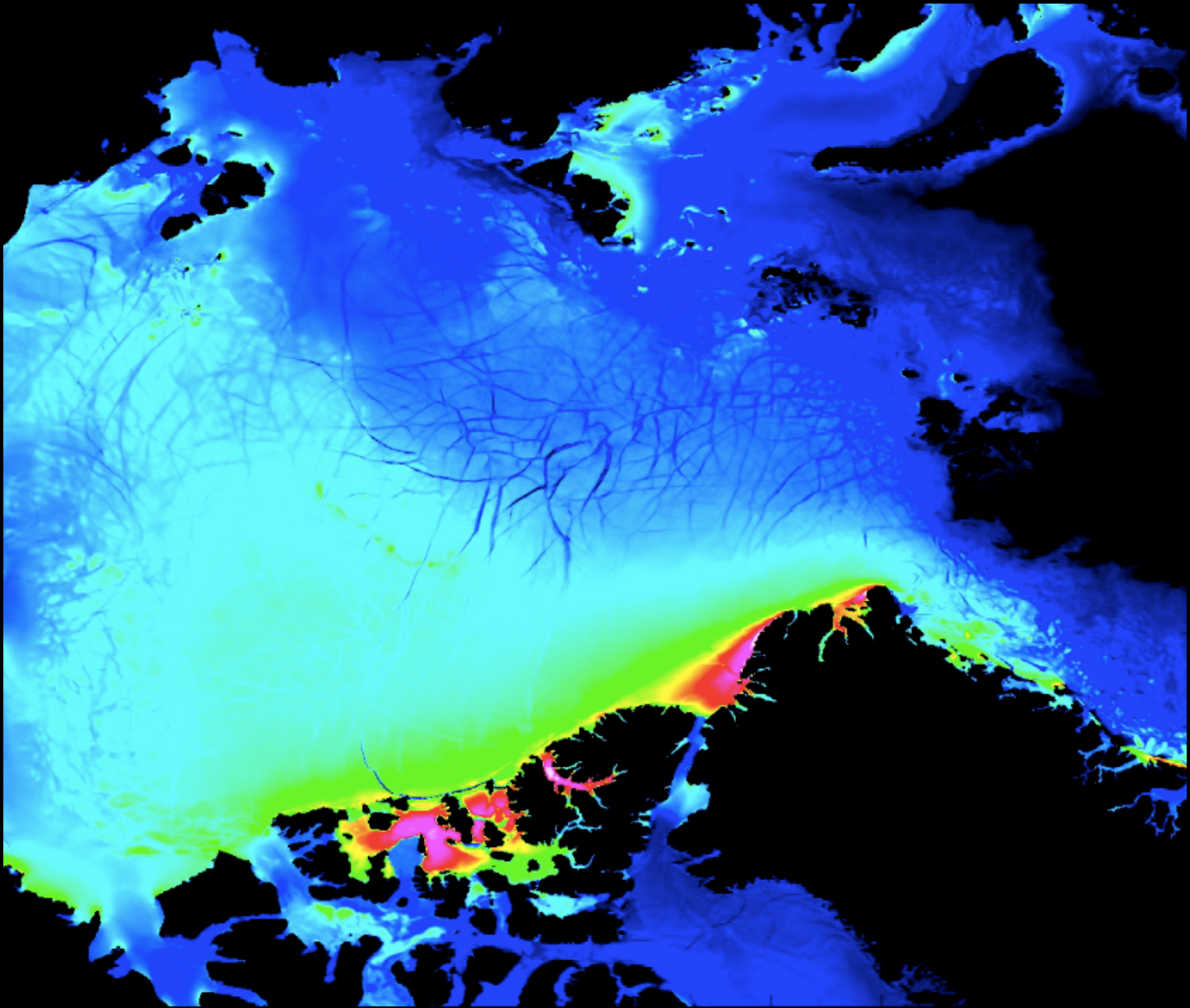
Convergence and accuracy of sea ice dynamics solvers

Martin Losch (AWI)
Annika Fuchs (AWI), Jean-François Lemieux (EC)



Sea ice thickness with MITgcm (JPL)

Review of 2D momentum equations

$$m\frac{D\mathbf{u}}{Dt} = -mf\mathbf{k}\times\mathbf{u} + \boldsymbol{\tau}_{air} + \boldsymbol{\tau}_{ocean} - m\nabla\phi(0) + \mathbf{F}, \qquad (1)$$

$F_j = \partial_i\sigma_{ij} =$ divergence of symmetric stress tensor of rank 2

Viscous-Plastic (VP) constitutive law (rheology):

$$\sigma_{ij} = 2\eta\dot{\epsilon}_{ij} + [\zeta - \eta]\,\dot{\epsilon}_{kk}\delta_{ij} - \frac{P}{2}\delta_{ij}. \qquad (2)$$

EVP equations (NOT intended to be a different rheology):

$$\frac{1}{E}\frac{\partial\sigma_{ij}}{\partial t} + \frac{1}{2\eta}\sigma_{ij} + \frac{\eta-\zeta}{4\zeta\eta}\sigma_{kk}\delta_{ij} + \frac{P}{4\zeta}\delta_{ij} = \dot{\epsilon}_{ij}. \qquad (3)$$

$$\dot{\epsilon}_{ij} = \frac{1}{2}(\partial_i u_j + \partial_j u_i) = \text{strain rates}$$

$$P = P^* hc \cdot e^{-C^*(1-c)} \qquad \zeta = \min\left(\frac{P}{2\Delta}, \zeta_{\max}\right) \qquad \eta = \frac{\zeta}{e^2}$$

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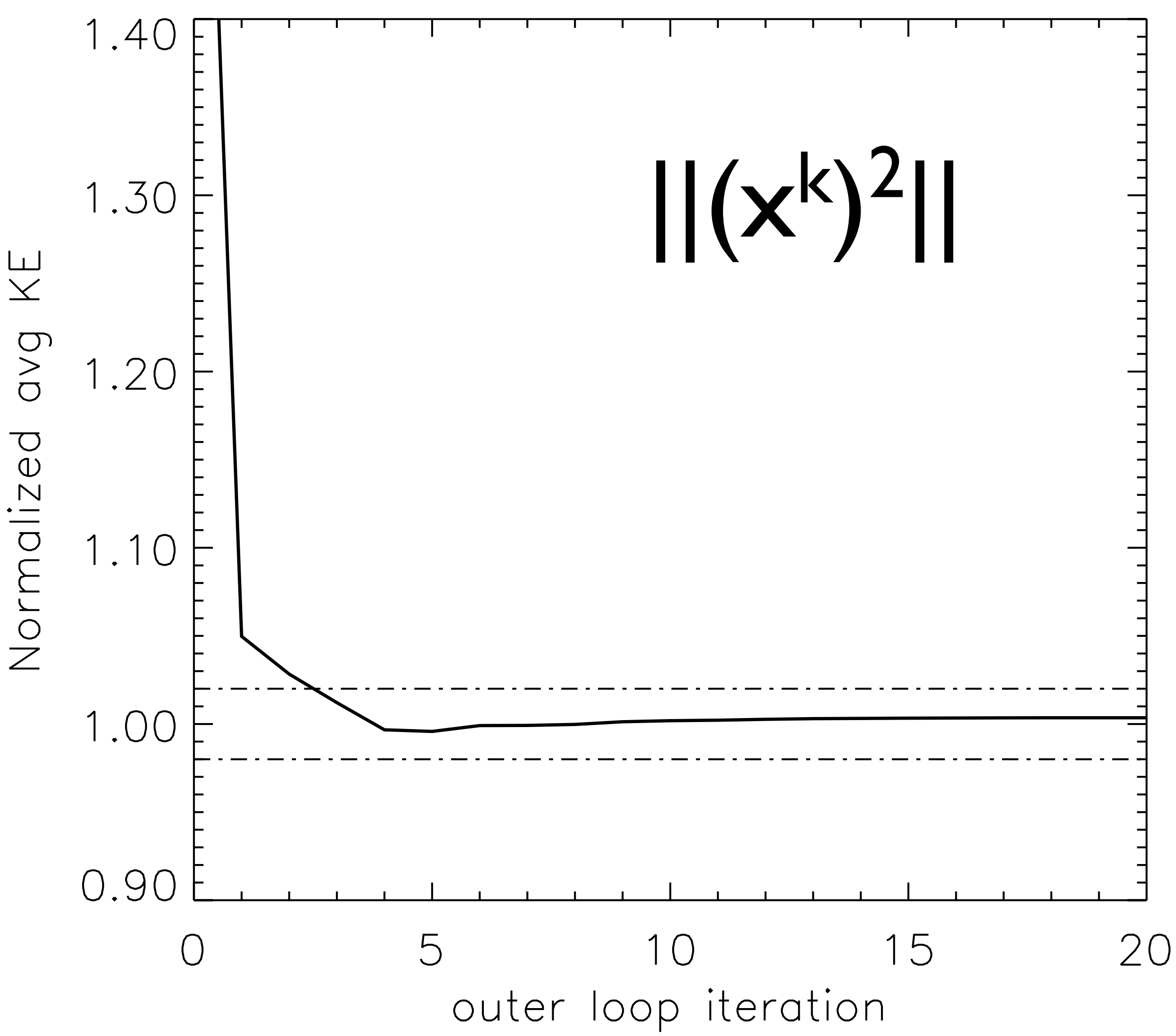
classical implicit VP-solver: the Picard solver

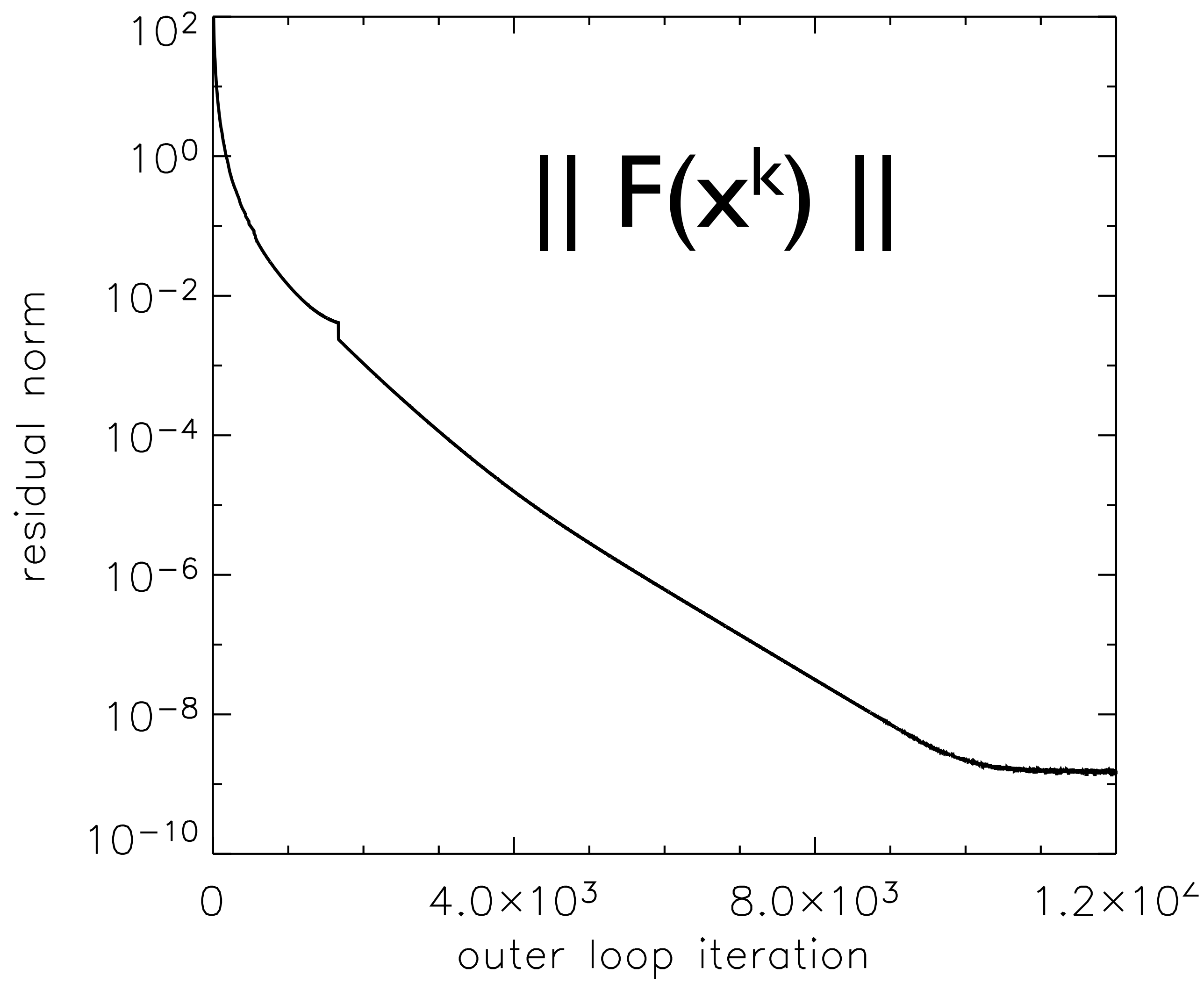
for $A(x)\ x = b(x)$

$A(x^{k-1})\ x^{k-1} - b(x^{k-1}) = F(x^{k-1})$

outer (non-linear) loop:

until (some criterion), solve $A(x^{k-1})\ x^{k} = b(x^{k-1})$





Lemieux and Tremblay (2009)

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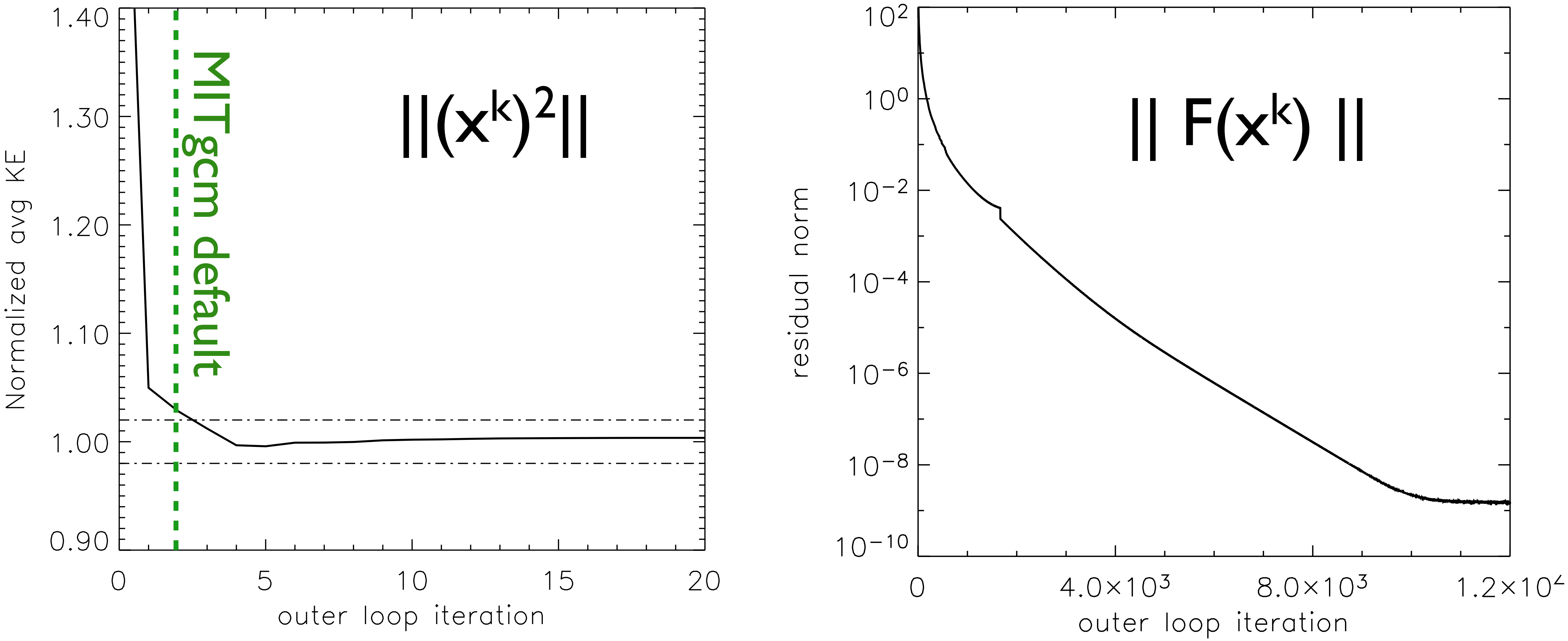

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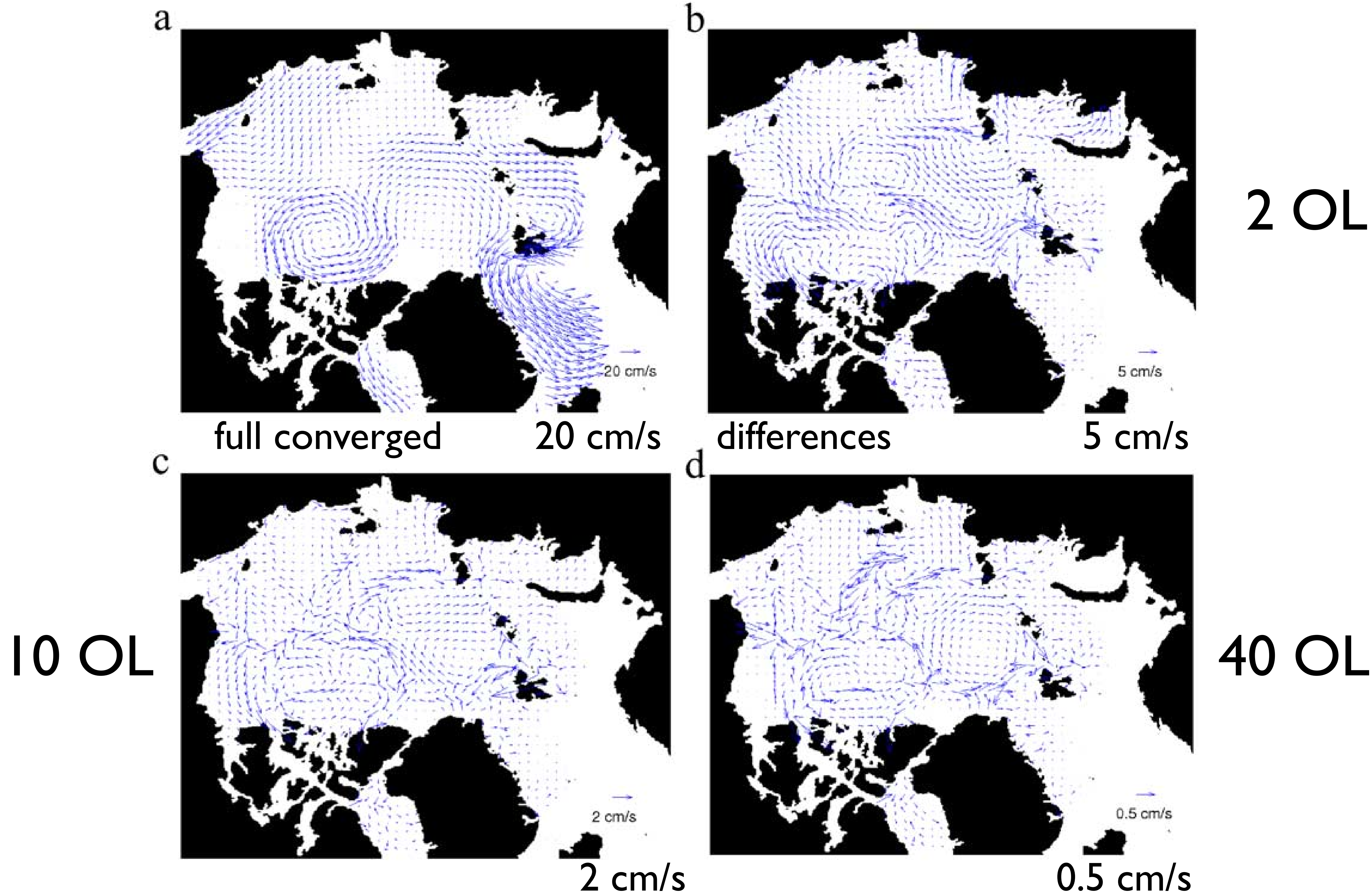


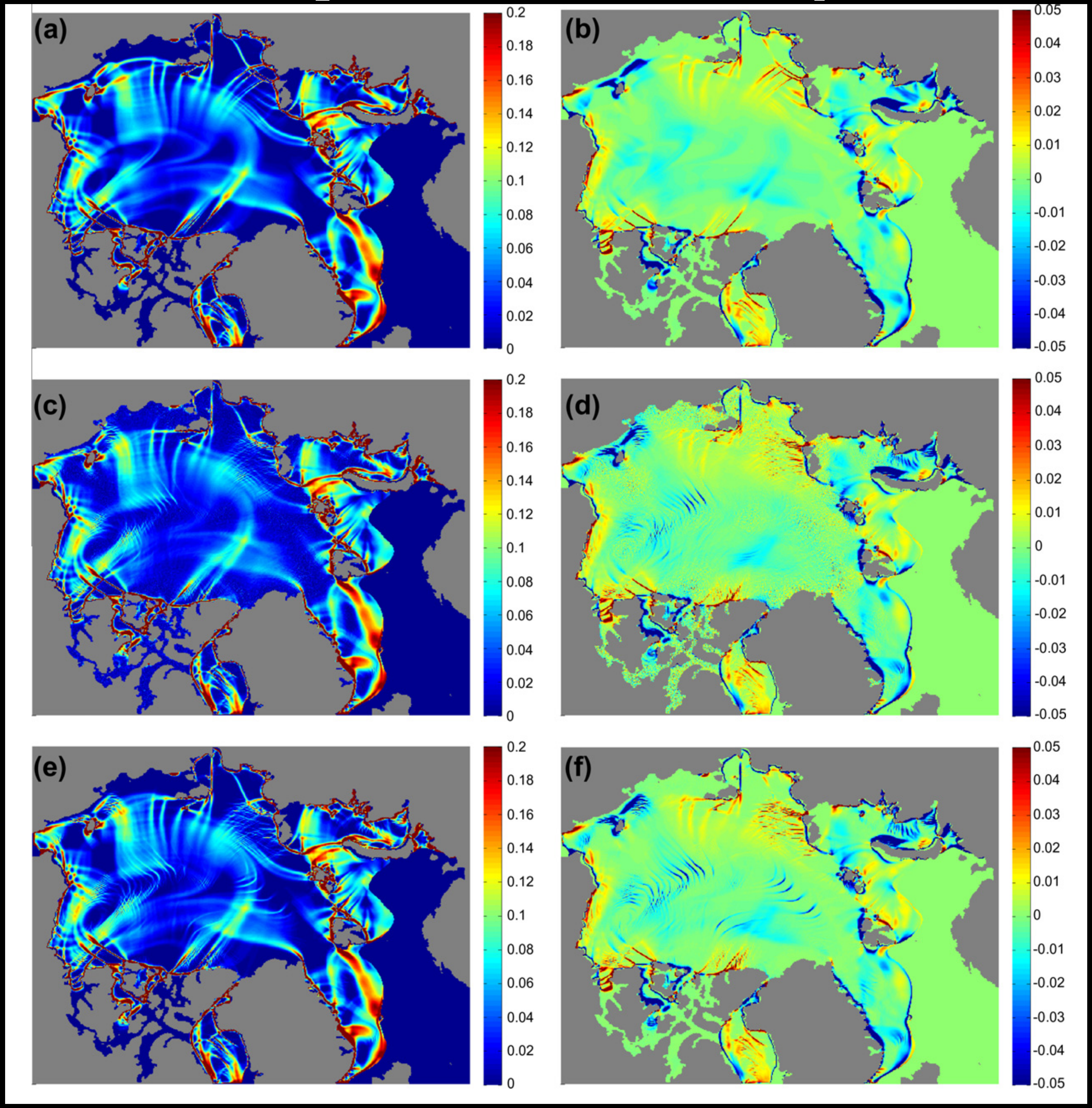
Lemieux and Tremblay (2009) (for long time steps)

EVP isn't any better (but faster)

reference

EVP, 120 sub-cycles

EVP, 1980 sub-cycles



Lemieux et al. (2012), shear and divergence (per day)

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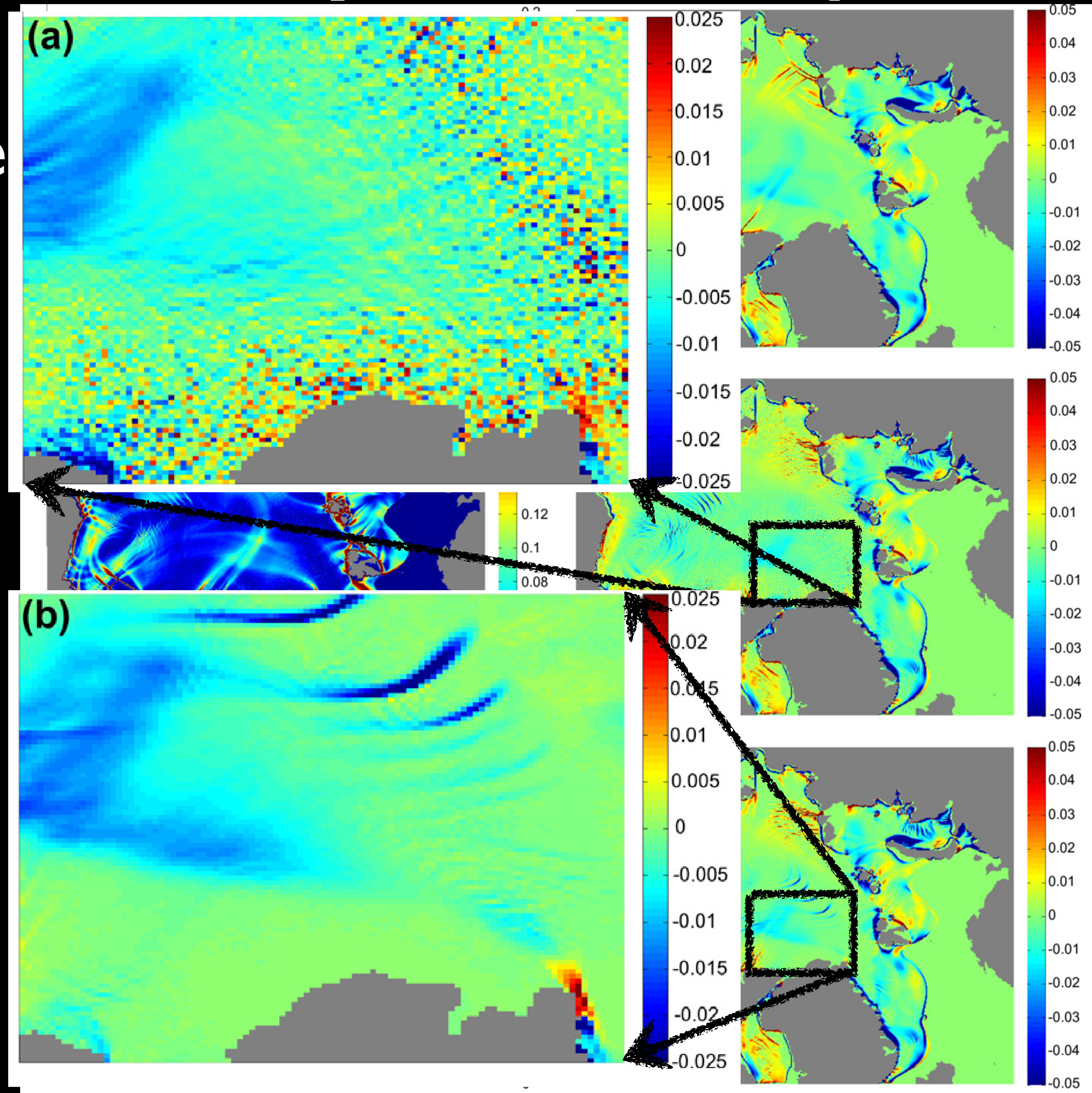

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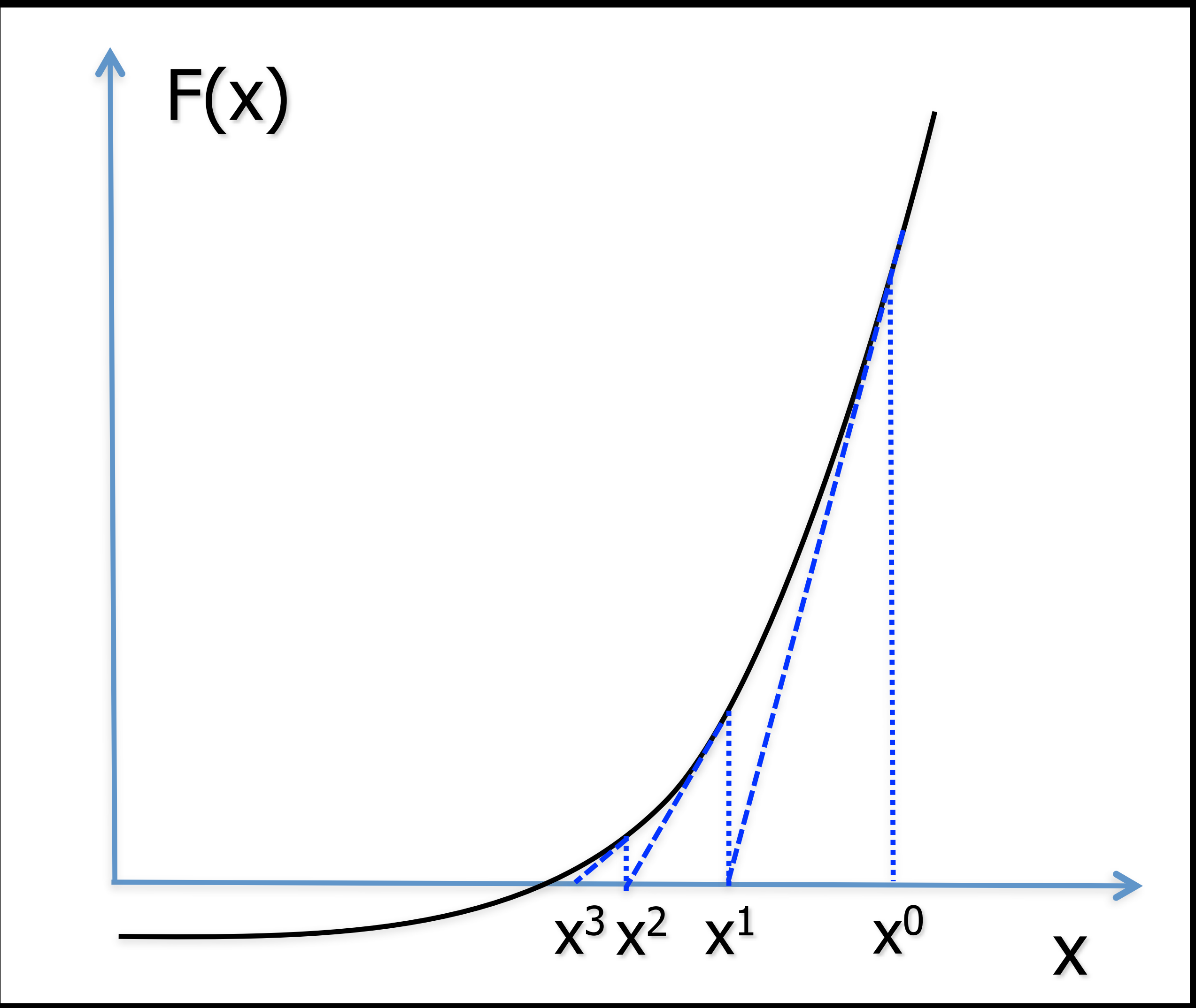
new to sea ice dynamics: the JFNK solver

$$[A(x^{k-1})\, x^k - b(x^{k-1}) = F(x^k)]$$

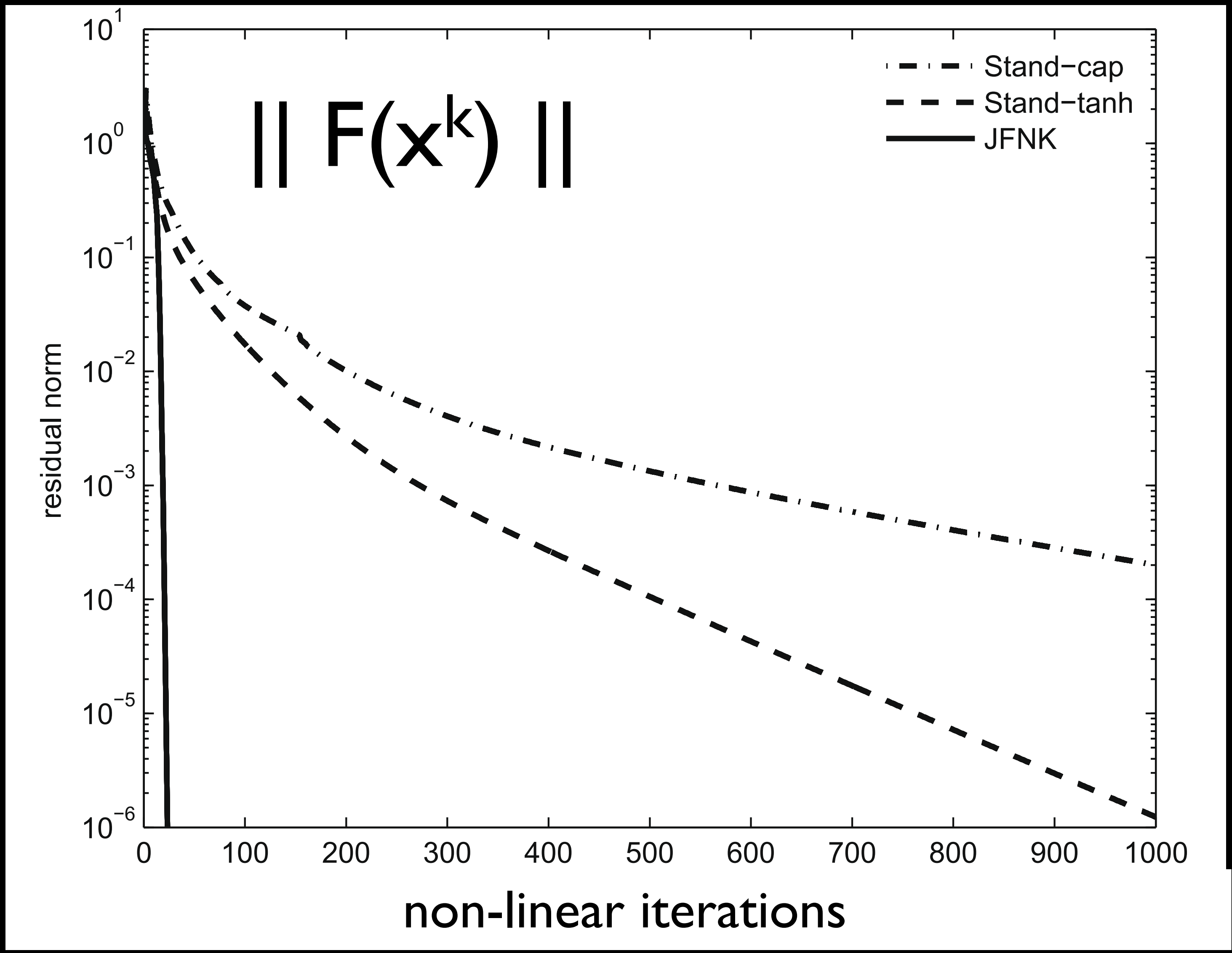
$$(0 =)\ F(x^{k-1}+\Delta x^k) = F(x^{k-1}) + F'(x^{k-1})\, \Delta x^k, \quad F' = J(\text{acobian})$$

until $\|\, F(x^k)\, \| < \text{tol}$, solve $J(x^{k-1})\Delta x^k = -F(x^{k-1})$

and update $x^k = x^{k-1} + \Delta x^k$



Newton method



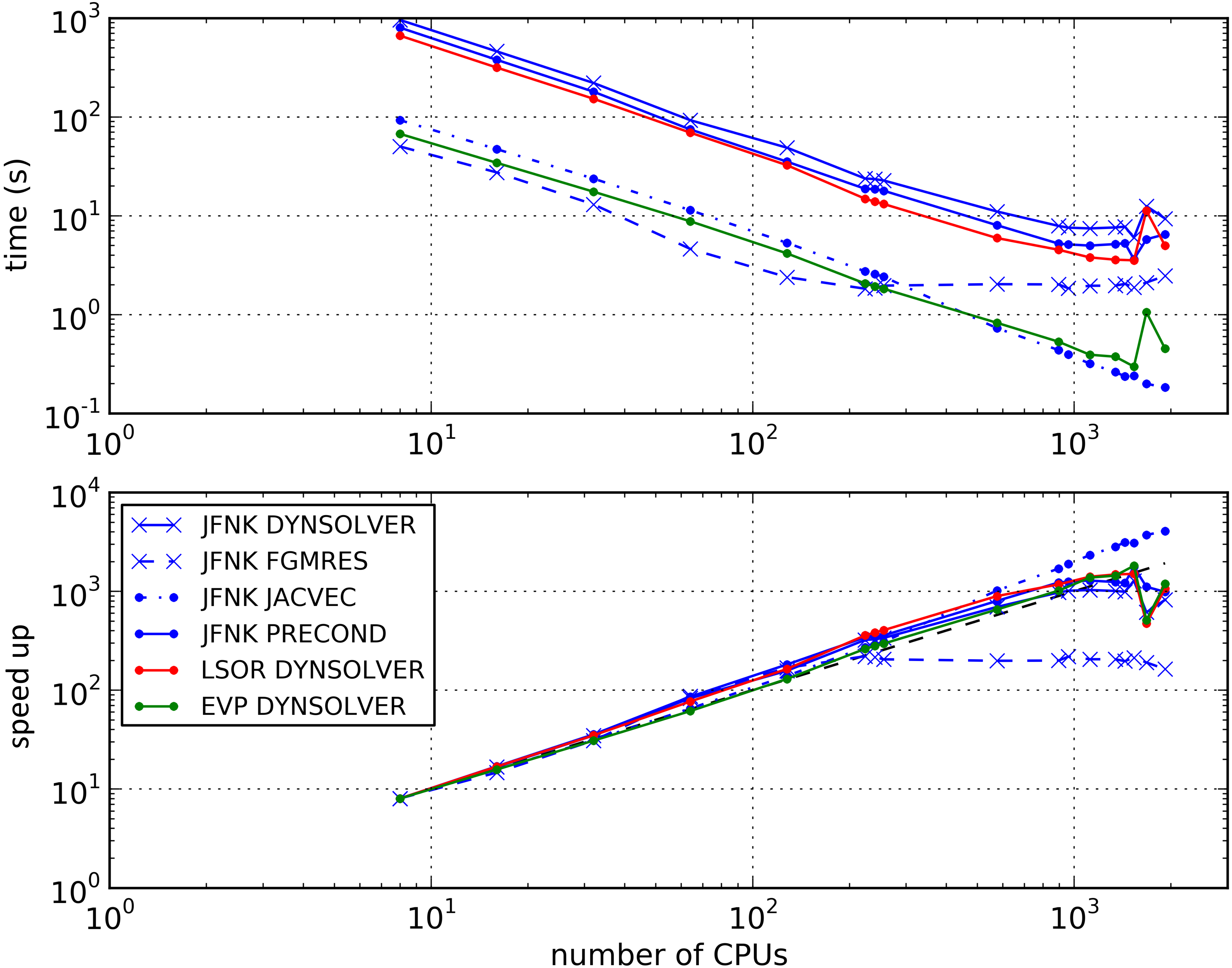
Lemieux et al. (2010)

JFNK in the MITgcm

- FGMRES with preconditioner (LSR, Zhang and Hibler, 1997)
- exact Jacobian times vector possible by AD
- parallel code:
  - scalar products in FGMRES
  - restricted additive Schwarz (RAS) method in LSR
- vector code:
  - iterative preconditioner (LSR)
  - coloring (zebra) method

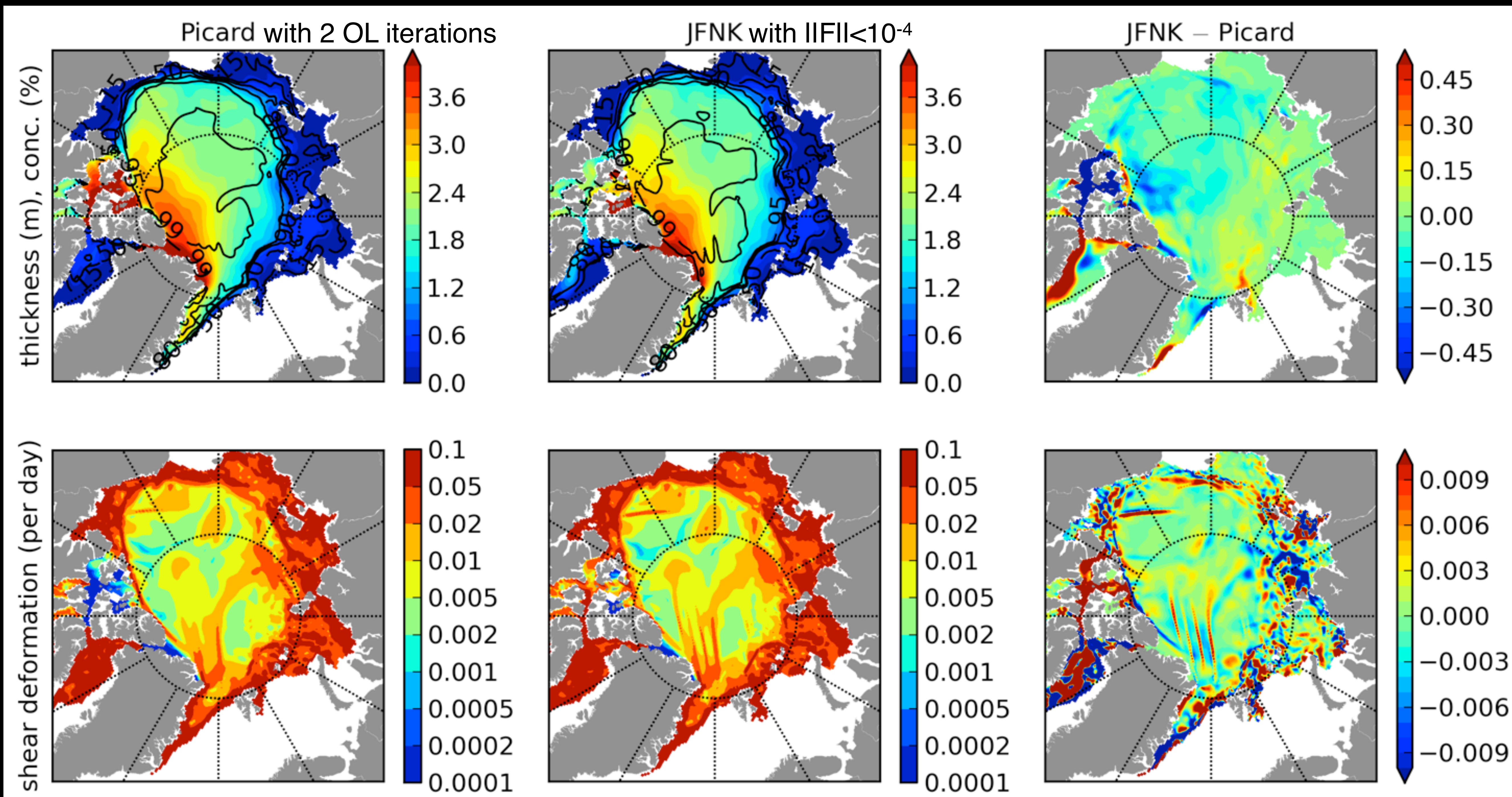
Losch et al. (2014)

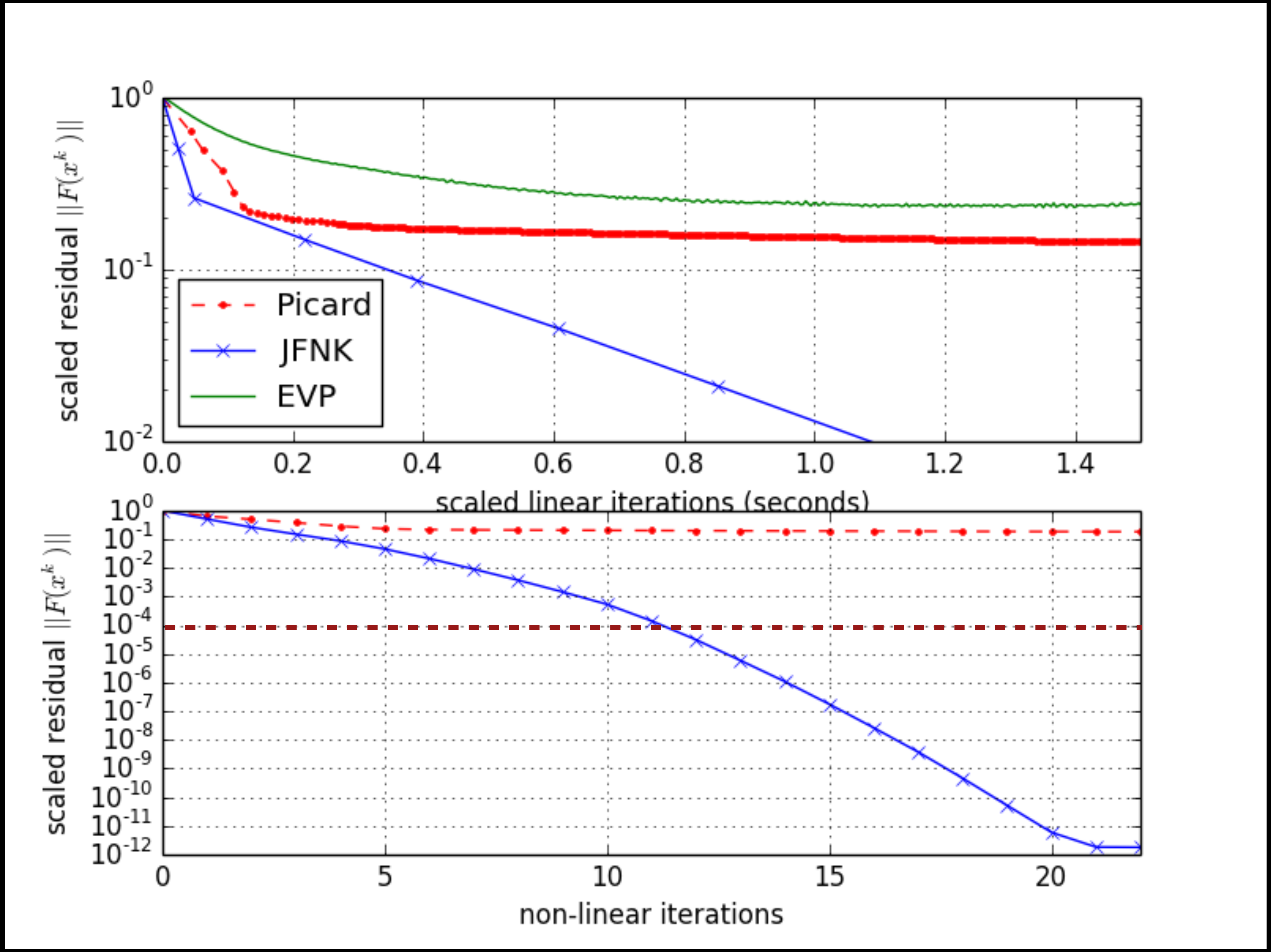
Parallel performance of solvers (4km Arctic configuration)



Does it matter?

after nearly 40 years of simulation: average of Oct, 1995





"Timing" of solvers

(Is JFNK really faster?)

Accurate solvers affect

- ice thickness distribution
- linear kinematic features
- computer time!!!

Summary

- traditional Picard solver converges slowly
- EVP solver does not converge at all (to VP)
- JFNK-solver is efficient but expensive

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JFNK-solver is available for

large-scale problems