Impact of advection schemes on restratification

Abstract
A host of studies has recognized that truncation errors of the discretized advection terms lead to spurious mixing and dissipation (Fig. 1) and may interact nonlinearly with turbulent mixing and transport. To investigate the impacts of spurious mixing and dissipation, we implemented some of the most novel advection schemes into the coastal ocean model GETM. We quantified spurious dissipation [Klingbeil, 2014] and mixing of the advection schemes (Fig. 3) in idealized experiments of baroclinic instabilities (Fig. 2) ranging from mesoscales (small Rossby number) to sub-mesoscales (order-one Rossby number). The processes at submesoscales are distinct from mesoscale by their contribution to restratification of the mixed layer. Such analyses (Fig. 4) help to choose between highly accurate but complex schemes and lower-order less complex schemes balancing accuracy and computational costs. The major outcome of the present study is that both, numerically induced dissipation (leading to a decrease of kinetic energy) and numerically induced mixing (leading to an increase of background potential energy), artificially delay the restratification process [Mohammadi-Aragh, 2015], an effect that needs to be taken into account if parameterizations for eddy-induced mixing and dissipation are compared with numerical model simulations.

Fig. 1: Advection schemes introduce numerical diffusion. How do we quantify it?

\[ ND = \int \frac{1}{2} \chi_{x} \rho_{0} dV \, dt \]

\[ \chi_{x} = - \left( \frac{\text{ADV}(u_{x})^{2} - \text{ADV}(u_{y})^{2}}{\Delta t} \right) \]

ADV is the advection operator.

Fig. 2: Experiment: Baroclinically unstable front produces eddies which restratify the front. How large is numerical dissipation compared to mechanical energy?

(a, c, e): Horizontal surface temperature, and (b,d,f): Zonally averaged temperature.

Fig. 3: Evolution of energy distribution during restratification process.

Fig. 4: Restratiﬁcation with different advection schemes and resolutions.

<table>
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<th>Advection schemes</th>
<th>SPL-1/3 and Superbee introduce the maximum positive and negative numerical diffusion, respectively.</th>
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<td>SPL-1/3</td>
<td>WENO performs better than most TVD schemes.</td>
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<td>SPL-max-1/3</td>
<td>MP5 performs best regarding the conservation of Energy.</td>
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<td>Third-Order-Upwind</td>
<td>Increasing resolution reduces numerical diffusion.</td>
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Conclusions
1. Numerical dissipation is more responsible for the energy loss than numerical mixing.
2. Dissipative advection schemes slow down stratiﬁcation process.
3. Different advection schemes affect restratiﬁcation especially in low Ro regime.
4. Modern schemes like WENO or MP5 reduce artiﬁcial numerical mixing and thus improve the simulation of eddy restratification.
5. SPL-max-1/3 is the best energy conserving TVD scheme.

References