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A Law for Small Scale, Continuous Calving

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Ice shelves are formed by the viscous flow of inland ice into the ocean, they are floating and loosing mass by iceberg calving. There are two different kinds of calving: large tabular icebergs detach as singular events in time, and small scale calving occuring on a rather continuous time scale. Three visco-elastic approaches are discussed, in order to derive a general law for calving rates applicable to small scale calving. The results are highly dependent on the termination criterium for each approach, hence the computed calving rate has to be adapted and validated with measurements to get the most qualified value.

1 Introduction

Ice reacts on a short time scale as an elastic solid, and on the long time scale as a viscous fluid. Therefore, a visco-elastic material model is most suitable to describe the behavior of an ice shelf. The results of stresses and strains near the ice front for two different material models are presented: The Kelvin-Voigt model, describing a solid, and the Maxwell model, describing a fluid behavior. A three dimensional linear elastic model shows that the results in a cross section along the flow line of an ice shelf are independent of the side boundary conditions, if the ice shelf is wide enough. Consequently, the ice shelf is modeled as a two dimensional rectangular block, assuming plane strain conditions. The block is loaded by gravity and surface loads. The boundary conditions are on the left the water pressure at the ice front, at the bottom the buoyancy force of the water, where h_{sw} denotes the ice draft and w the displacement in z direction, and on the right a constant displacement u to describe the constant flow over the thickness in an ice shelf, see Fig 1. The ice is assumed to be homogeneous and isotropic.

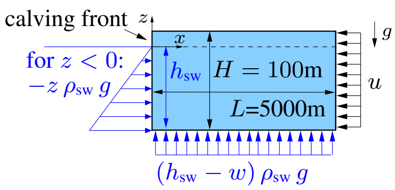

Fig. 1: Boundary conditions

2 Visco-elastic analysis

For the visco-elastic analysis, the stress and strain tensors are split into a volumetric part and into a deviatoric part

$$\boldsymbol{\sigma} = \frac{1}{3}\,\mathrm{tr}(\boldsymbol{\sigma}) + \boldsymbol{\sigma}^D, \qquad \boldsymbol{\varepsilon} = \frac{1}{3}\,\mathrm{tr}(\boldsymbol{\varepsilon}) + \boldsymbol{\varepsilon}^D. \tag{1}$$

Elastic deformation is allowed for the volumetric change and only the deviatoric stress is influenced by the visco-elastic response, for details see [1]. The rheological model of a Kelvin-Voigt element consists of a spring and a dashpot in parallel. If a Kelvin-Voigt material is suddenly loaded by a constant stress, this is initially taken up by the dashpot. The stress redistributes from the dashpot to the spring in time. After a certain time period, the strain of the system, which increases exponentially at a constant stress, will asymptotically converge to a maximum value, the value of the linear elastic case. In contrast, the rheological model of the Maxwell element is composed of a spring and a dashpot in series. Then the short term deformations effect a pure elastic answer. The elastic answer decays exponentially with time, consequently the viscous stress remains as the longterm answer. Therefore, the equations for the stress tensor $\boldsymbol{\sigma}$ and the deviatoric stress computed with the Kelvin-Voigt model $\boldsymbol{\sigma}^D_{\mathrm{KV}}$, or the Maxwell model $\boldsymbol{\sigma}^D_{\mathrm{M}}$, respectively, are as follows

$$\boldsymbol{\sigma} = \frac{E}{3(1-2\nu)}\,\mathrm{tr}(\boldsymbol{\varepsilon}) + \boldsymbol{\sigma}^D \quad \text{with} \quad \boldsymbol{\sigma}^D_{\mathrm{KV}} = 2G\boldsymbol{\varepsilon}^D + 2\eta\dot{\boldsymbol{\varepsilon}}^D \quad \text{or} \quad \boldsymbol{\sigma}^D_{\mathrm{M}} + \frac{\eta}{G}\dot{\boldsymbol{\sigma}}^D_{\mathrm{M}} = 2\eta\dot{\boldsymbol{\varepsilon}}^D \tag{2}$$

with the shear modulus $G = \frac{E}{2(1+\nu)}$. The material parameters of ice for this study are the Young's modulus $E = 9\,$GPa, the Poisson's ratio $\nu = 0.325$ and the viscosity $\eta = 10^{16}$ Pa·s. For further information see [2].

3 General procedure

The stress disturbance at the ice front leads to tensile stresses at the upper part and compression at the lower part near the front. The position of the highest stress, resp. strain is crucial to indentify the most probable position for calving and is located on the surface. Therefore, only the results on the surface are subsequently considered. Different approaches introduce three possibilities of the termination criteria. If this is reached, an iceberg calves off, see Fig. 2. After a calving event the initial conditions of the remaining ice shelf are taken from the deformation and stress state prior to the calving event. The water pressure suddenly has an effect on the new ice front and the stress/strain evolution at the surface is analyzed over time.

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The calving rate $c^{\perp}$ in both models is computed by

$$c^{\perp} = v_{\text{flow}} - v_{\text{loss}} \qquad \text{with} \qquad v_{\text{loss}} = \frac{\lambda}{t_{\text{c}}} \tag{3}$$

where v_{flow} is the measured flow velocity of the ice shelf, λ is the distance of the maximum stress to the ice front, see Fig. 2, and t_c is the termination time, when the failure criterion is reached. If the so defined calving rate is larger than 0, the ice shelf extends over time, if it is 0, the ice shelf is stable and otherwise if the calving rate is smaller than 0, the ice shelf retreats.

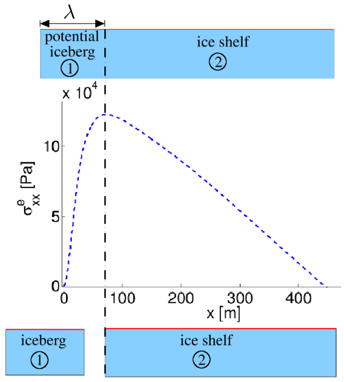

Fig. 2: Calving event

4 Results

In the first approach a termination criterion using a critical stress of 0.33MPa, discussed in [3], is applied. If the stress exceeds this value the next iceberg breaks off. The left panel of Fig. 3 shows the stress evolution after a calving event at the surface near the ice front over time. The crosses denote the maximum tensile stress of the ice shelf. The right panel in Fig. 3 illustrates these maxima over time. This termination criterion is unrealistic, because it is only valid for a small thickness interval due to the highly dependency of the stress to the thickness. If the thickness H is below 270 m the critical strength is never reached and if H is larger than 340 m than the maximum tensile stress is always above the critical value.

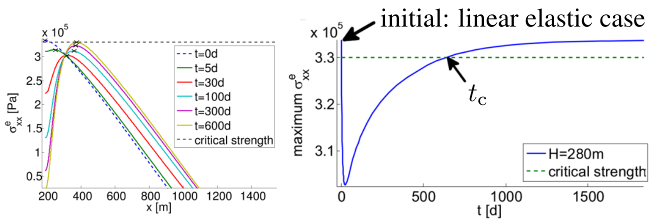

Fig. 3: Kelvin-Voigt material model with the stress criterion

Therefore, a second approach is developed to be valid for the whole thickness range of Antarctic ice shelves. Due to the convergence of the Kelvin-Voigt model to the linear elastic case after time, consider Fig. 4, the difference of the stress maxima of two successive time steps has to be smaller than a certain termination bound. This approach leads to more realistic results than the first one. Nevertheless the convergence bound has a crucial influence on the calving rate.

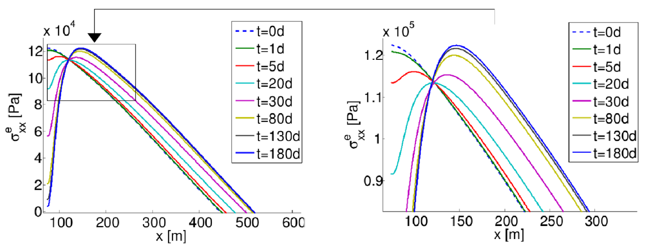

Fig. 4: Kelvin-Voigt material model with the self-similarity criterion

The last approach deals with a fluid material model, the visco-elastic Maxwell model. For this model the stresses are the highest directly after a calving event and a critical strain criterion have to be applied. The critical strain values are discussed in [4]. Figure 5 displays the strain evolution in time between different calving events.

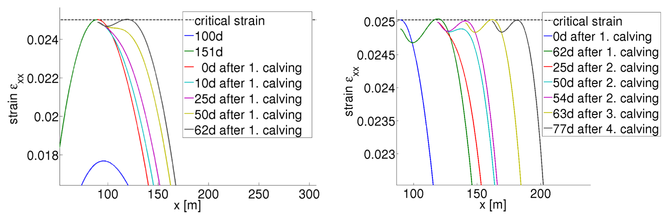

Fig. 5: Maxwell material model with the strain criterion

The time intervals between two calving events are smaller as for the Kelvin-Voigt solid, but also the iceberg length is smaller. This approach is also valid for all thicknesses, but has a small sensitivity of the maximum strain w.r.t. time. The calving rates are nearly the same for the second and last approach and have to be adapted and validated with measurements.

5 Conclusion

The different visco-elastic material approaches are mainly dependent on the termination bound. Therefore, the adaptation and validation of these parameters due to measurements are necessary. The observation of small scale calving is until now still limited. Nevertheless more and more satellite images provide images with higher resolutions and higher coverage.

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