A Comparison of Data Assimilation with the Ensemble Kalman Filter and the SEEK Filter applied to non-linear Shallow Water Equations

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The Problem
During the last years there has been an extensive development of stochastic filtering algorithms based on the Kalman filter intended for application to high-dimensional numerical models. Of those filters, we directly compare two widely used algorithms: the Ensemble Kalman filter (EnKF) and the Singular Evolutive Extended Kalman filter (SEEK). In addition we consider the Singular Evolutive interacting Kalman filter (SEIK), which can be regarded as an interpolated version of the SEEK algorithm or as an ensemble filter using a preconditioned ensemble. The comparison focuses on the mathematical foundations of the algorithms and their numerical requirements as well as on their application to a model ocean. In train experiments with synthetic observations of the surface elevation the assimilation behavior of the algorithms is assessed. The computational burden and filter performance depend strongly on the ensemble size and rank of the state covariance matrix. Hence the ensemble size and the rank are used as a parameter in the experiments.

Filter Algorithms

**EnKF** The Ensemble Kalman Filter applies a Monte-Carlo method to solve the Fokker-Planck equation governing the evolution of the statistics of a stochastic model.

**SEEK** The Singular Evolutive Extended Kalman Filter is derived from the Extended Kalman Filter by approximating the state error covariance matrix by a matrix of reduced rank and evolving this matrix in decomposed form.

**SEIK** The Singular Evolutive Interpolated Kalman Filter was originally derived from the SEEK algorithm. Alternatively it can be interpreted as a reduced-rank-preconditioned ensemble Kalman filter.

Assimilation Experiments

**Configuration** For the experiments we generated synthetic observations from a model run by disturbing the surface elevation by normally-distributed noise.

The initial state estimate was chosen as the mean state of a model run over 40000 time steps. The state covariance matrix was computed as the variation of this state sequence about the mean. An incomplete eigenvalue decomposition of this matrix, retaining only the largest eigenmodes, we generated the low-rank approximation for use with the SEEK and SEIK algorithms. The analysis step was performed after each 1000 time steps.

Filtering
To relate the filter performance to the computational burden, all three algorithms were configured in such a way that each algorithm required the same number of model evaluations. In addition, we implemented the algorithms to achieve minimum computing times.

For large ocean models the number of model evaluations is usually quite restricted due to limited computing power and time. Therefore we tested the filter performances for a small ensemble of size 21. Additionally, we performed experiments with a large ensemble of size 201, which is expected to provide a much better representation of the error statistics.

Summary
For the experiments presented here the SEEK algorithm shows superior filter performance for the small ensemble and is comparatively fast. The SEIK filter is faster than the SEEK but yields better performance only for larger ensembles. The EnKF is expected to be fastest for very large ensembles. It shows a filter performance similar to that of the SEIK.

Computation Time
The model evaluations take more than 95% of the computing time. Since the number of model evaluations is equal for all three filters, we consider only the computing time of the filter.