Jörn Behrens [Joern.Behrens@awi.de], Alfred Wegener Institute for Polar and Marine Research, Bremerhaven, Germany

Adaptive Atmospheric Modeling on Unstructured Grids - Generic Techniques

Abstract

Efficient unstructured grid adaptive modeling requires for advanced programming and numerical techniques. In this presentation we demonstrate several such generic techniques applied to simple example applications in atmospheric and ocean modeling. Efficient grid generation is achieved by triangular bisection, optimized data locality can be realized by space-filling curve ordering, and data management is stream lined by a gather-scatter paradigm.

We apply these numerical schemes to a semi-Lagrangian cell integrated mass conserving advection scheme, and to a finite-element wave propagation application in tsunami ocean modeling.

Triangular refinement

- Bisection of marked edge.
- Applicable in 2D (triangles) and 3D (tetrahedra), see fig. 1.
- Generic refinement tree: binary tree data structure.
- Linearization of tree structure by space-filling curves (see below).

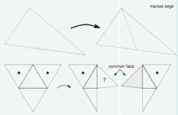

Fig. 1 Bisection refinement; top: in 2D, bottom: in 3D

Data Management

- Gather-scatter paradigm (see fig. 2).
- Object oriented, tree structured data for mesh management.
- Vector structured data for numerical computations.
- Locality preservation via space-filling curves (see below).
- Low overhead (<1%) for gather and scatter operations.

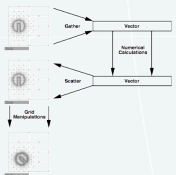

Fig. 2 Gather-scatter paradigm

Space-filling Curves (SFC)

- Computation of SFC indices "on the fly" in combination with refinement strategy (see above).
- Zero overhead for SFC computation and reordering.
- SFC indexing: bit manipulations based on refinement level, orientation, mother index (fig. 6).
- Domain partitioning for parallelization: optimal load balancing, near optimal edge cut (fig. 7).
- Matrix structure optimization by reordering unknowns (comparison of different algorithms in fig. 8).
- Cache optimization due to neighborhood preservation property of SFC (fig. 9)

Fig. 6 bit patternrelated to SFC odering

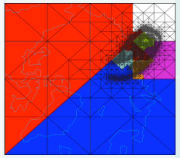

Fig. 7 Domain partitioned by space-filling curve

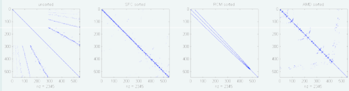

Fig. 8 Matrix structure with different ordering algorithms; left to right: unsorted, SFC, Cuthill-McKee, minimum degree

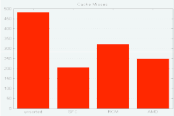

Fig. 9 Cache misses with different orderings

Implementation/Software

- Fortran 90 library with module interface.
- Modular software package, simple API.
- Open source (after registration) with documentation.
- Web page for ticketing, Wiki, etc.:

  http://www.amatos.info

- User feedback is welcome!

amatos
the grid generator

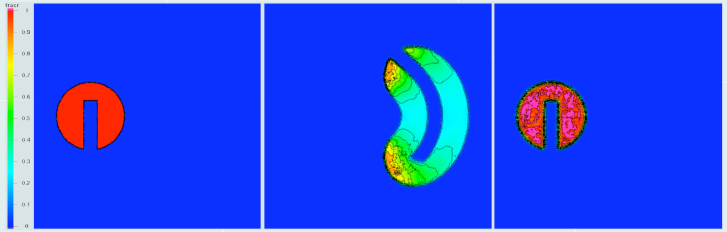

Fig. 10 Test case with accelerating and decellerating wind field; left: initial state, center: half revolution, right: full revolution

Cell-integrated semi-Lagrangian Advection

- Integral form of advection equation: $\frac{d}{dt}\int_{V(t)} \rho \; dx = 0$
- Semi-Lagrangian time discretization of integral form:

$$\int_{V(t)} \rho(\mathbf{x}, t) \; dx = \int_{V(t-\Delta t)} \rho(\mathbf{x} - \alpha, t - \Delta t) \; dx$$

- Representation of reference volume by dual cell (fig. 11)
- Geometric intersection of upstream dual mesh with old mesh

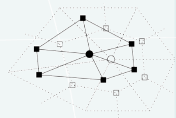

Fig. 11 Dual mesh (dotted with square corners) and upstream dual cell (solid)

Prototype Tsunami Model

- Based on shallow water equations:

$$\frac{\partial h}{\partial t} + \nabla \cdot [(h + H)\mathbf{v}] = 0$$

$$\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{v} + f(\mathbf{e}_r \times \mathbf{v}) = -g \nabla h + F$$

- Finite element discretization with P^1-P^1_{NC} elements (fig. 12)
- Radiation/reflection boundary conditions
- Second order leap-frog time stepping
- Refinement control by gradient
- Remeshing in each time-step (fig. 13)

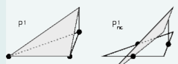

Fig. 12 Triangular linear conforming and linear non-conforming finite lement

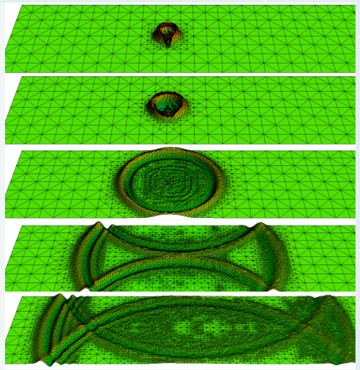

Fig. 13 Wave propagation in a prototypical tsunami model

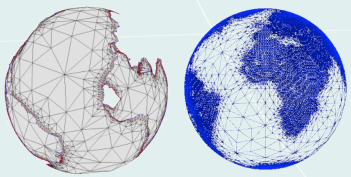

Fig. 3 Mesh of world ocean, and global mesh with refinement along topography gradients.

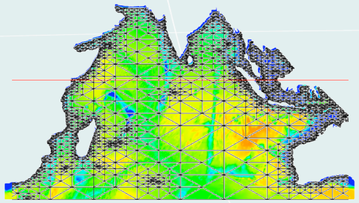

Fig. 4 Automatically generated refined mesh of Indian Ocean

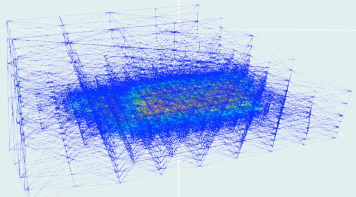

Fig. 5 Three-dimensional tetrahedral grid with local

Summary and Acknowledgements

Efficient data structures and strategies for 2D and 3D adaptive mesh refinement were introduced. These techniques are readily combined with advanced methods for atmospheric and ocean modeling.

Thanks to: F. Klaschka, L. Mentrup, S. Danilov, D. Sein, O. Kunst, M. Käser, J. Zimmermann.

Literature

- E. Bänsch (1991): Local Mesh refinement in 2 and 3 dimensions, *Impact of Comput. In Sci.*, **3**, 181–191.
- J. Behrens (2001): Parallelizing an Adaptive Dynamical Grid Generator in a Climatological Trace Gas Transport Application, in: Sorevik et al. (Eds.) *PARA 2000 proceedings*, LNCS **1947**, Springer Verlag, 170-176.
- J. Behrens (2005): Multilevel optimization by space-filling curves in adaptive atmospheric modeling. In Hülsemann et al. (Eds.) Frontiers in Simulation, SCS Publ. House, 186–196.
- J. Behrens (2006): *Adaptive Atmospheric Modeling*, Springer Verlag, LNCSE **54**, Berlin/Heidelberg.