# An unstructured adjoint tidal model, sensitivities and parameter optimization

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# Outline

Goal: Unstructured tidal model with inverse estimation of parameters; adjoint model generation via automatic differentiation

# Status of tidal modelling

- Regional or global simulations
- Tsunami and tidal wave interaction
- Internal tides
- Inclusion of tidal potential and LSA effect
- Wetting & drying
- Clamped or Flather open boundary condition
- Finite volume (FV) and finite element (FE) models on triangular, unstructured meshes
- Different time-stepping schemes

# Adjustment of parameters ...

... has to be done for each mesh:

- Bottom friction coefficient is an important energy sink, but largely unconstrained by measurements. It depends on bottom forms and roughnesses.
- Depth determines wave speed. (Bottom) topographic data is often a compilation of various data sources and may have smoothed features, that could be resolved with unstructured meshes.
- Open boundary values influence the solution significantly. They are given by model results itself and could therefore contain inconsistencies.

The number of parameters is too big to do manual adjustment of parameters.
$\rightarrow$ Inverse methods: They draw from the misfit between model results and observations informations about correct parameters.
We minimize a cost function by generation of an adjoint model with automatic differentiation and the BFGS algorithm.
The cost function is calculated after each tidal cycle:

$$J = \sum_{m=1}^{M} \left[ \left( B_m^{obs} - B_m^{mod} \right)^2 + \left( D_m^{obs} - D_m^{mod} \right)^2 \right] s_p \\ + \text{penaldepth} + \text{penalfriction}$$

$B$ resp $D$ are the real resp imaginary part of the oscillations. $s_p$ is a scaling coefficient. $M$ is the number of measurement points.

# Automatic differentiation ...

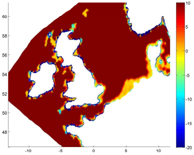
... is a technology for automatically adding statements for the computation of derivatives to computer programs.
www.autodiff.org
Advantages:

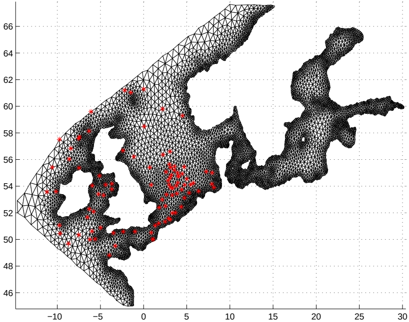
- Derivatives are accurate (contrary to FD methods)
- Adjoint model of the discretized equations
- Automatically generated adjoint models are easier to maintain
- Computation of Hessian for optimization algorithms is also possible
- Free software exists (TAMC, Tapenade, OpenAD,...)

# Test setup

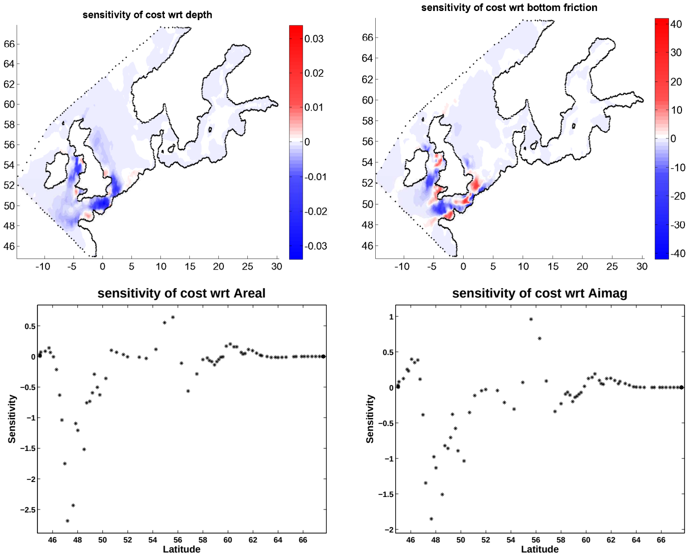
The adjoint model is generated of the explicit non-conforming FE code using TAMC.
(clamped boundary condition, no wetting and drying, minimal depth of 10m, no potential, only $M_2$ tidal forcing)

The scheme is tested on a very coarse mesh of the North- and Baltic Sea with only 7078 nodes. The cost function computes the misfit to 93 tidal gauges.

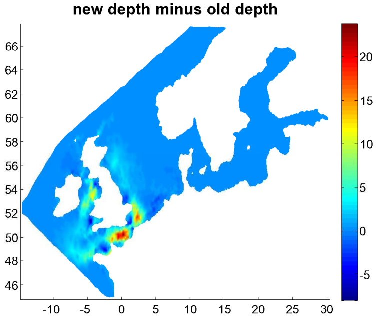


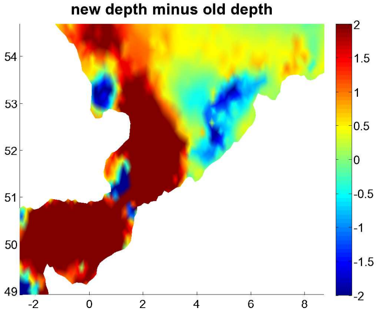


# Initial gradient of the cost function

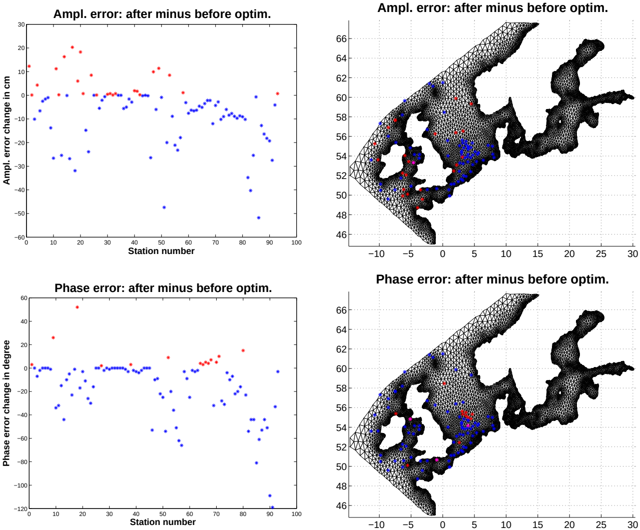


# Optimized depth with respect to tide gauges



# Reduction of error due to optimized bottom topography



Summary

- Model is more sensitive to changes in open boundary values than to bottom friction and depth.
- Optimized depth is consistent with our expectation
- Error reduction in more than two thirds of the stations.

Outlook

- Compare different AD tools to identify the most efficient
- Extend the simulations to finer resolving meshes and include more physical processes
- ...
- ... your suggestions ...

# Intercomparison of 2D unstructured tidal models

Comparison of 7 models with different spatial and temporal discretizations to simulate M2 tide in the North Sea. (Maßmann et al, 2009, Continental Shelf Research)

- Preference to certain FV and FE schemes
- Semi-implicit is faster, although solver needs to be called
- The order of approximation is determined by the spatial order
- With the same time step size FV is faster than FE methods
- A diagonal mass matrix makes lumping unnecessary and the code faster