ALFRED-WEGENER-INSTITUT
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Sea ice dynamics solvers in the MITgcm

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Complicated dynamics

which satellite?

Complicated dynamics

MITgcm (Menemenlis, Hill)

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Outline: Sea-ice solvers in MITgcm

- Picard solvers (LSR, Krylov)
- JFNK solver
- EVP solvers: mEVP, aEVP

- new MEB rheology in the pipeline

sea ice dynamics are very non-linear

$$m\frac{\partial \mathbf{u}}{\partial t} = \nabla \cdot \sigma + R, \qquad R = \text{other terms}$$

with $\sigma_{ij} = \frac{P}{2\Delta}\left\{2\dot{\epsilon}_{ij}e^{-2} + \left[(1-e^{-2})(\dot{\epsilon}_{11}+\dot{\epsilon}_{22}) - \Delta\right]\delta_{ij}\right\}$

with abbreviations

$$\Delta = \sqrt{(\dot{\epsilon}_{11}+\dot{\epsilon}_{22})^2 + e^{-2}\left[(\dot{\epsilon}_{11}-\dot{\epsilon}_{22})^2 + 4\dot{\epsilon}_{12}\right]}$$

$$\dot{\epsilon}_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) \qquad \text{(strain rates)}$$

$$\longrightarrow \quad m\frac{\partial \mathbf{u}}{\partial t} \propto \frac{\partial}{\partial x_i}\left(\frac{P}{\Delta}\frac{\partial u_i}{\partial x_j}\right) + \text{similar terms}$$

solution techniques: Picard method

$$\mathbf{A}(\mathbf{u}) \cdot \mathbf{u} = \mathbf{b}$$

$\Rightarrow$ solve $$\mathbf{A}(\mathbf{u}_{n-1}) \cdot \mathbf{u}_n = \mathbf{b}$$

- traditional method, e.g., PSOR, Hibler (1979), LSOR, Zhang and Hibler (1997), (Gauss-Seidel) for linear solver
- Krylov method for linear solver (Lemieux and Tremblay, 2009), requires preconditioner
- stable, but slow

solution techniques: JFNK solver

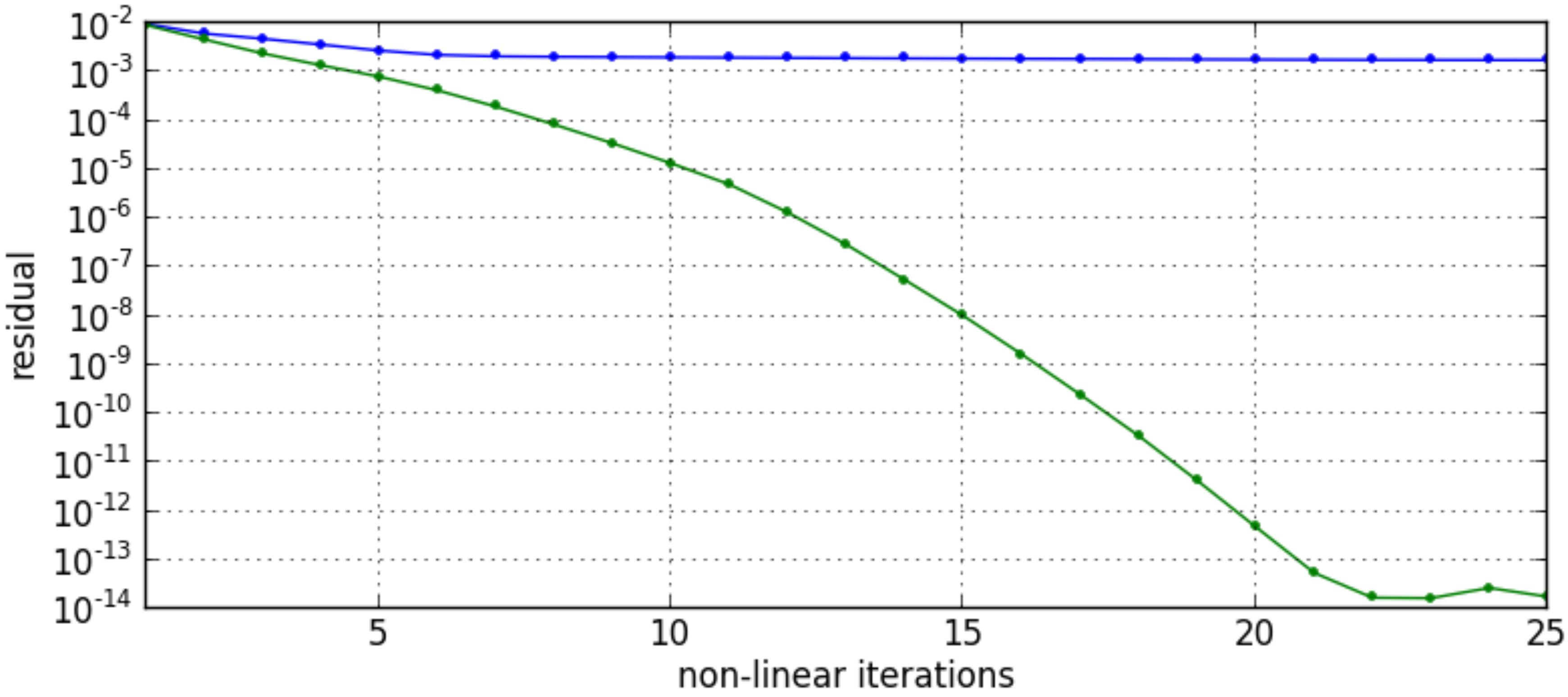
$$\mathbf{F}(\mathbf{u}) = \mathbf{A}(\mathbf{u}) \cdot \mathbf{u} - \mathbf{b}$$

$$\mathbf{F}(\mathbf{u}_n) = \mathbf{F}(\mathbf{u}_{n-1}) + \mathbf{F}'\Big|_{\mathbf{u}_{n-1}} \delta\mathbf{u} \stackrel{!}{=} 0$$

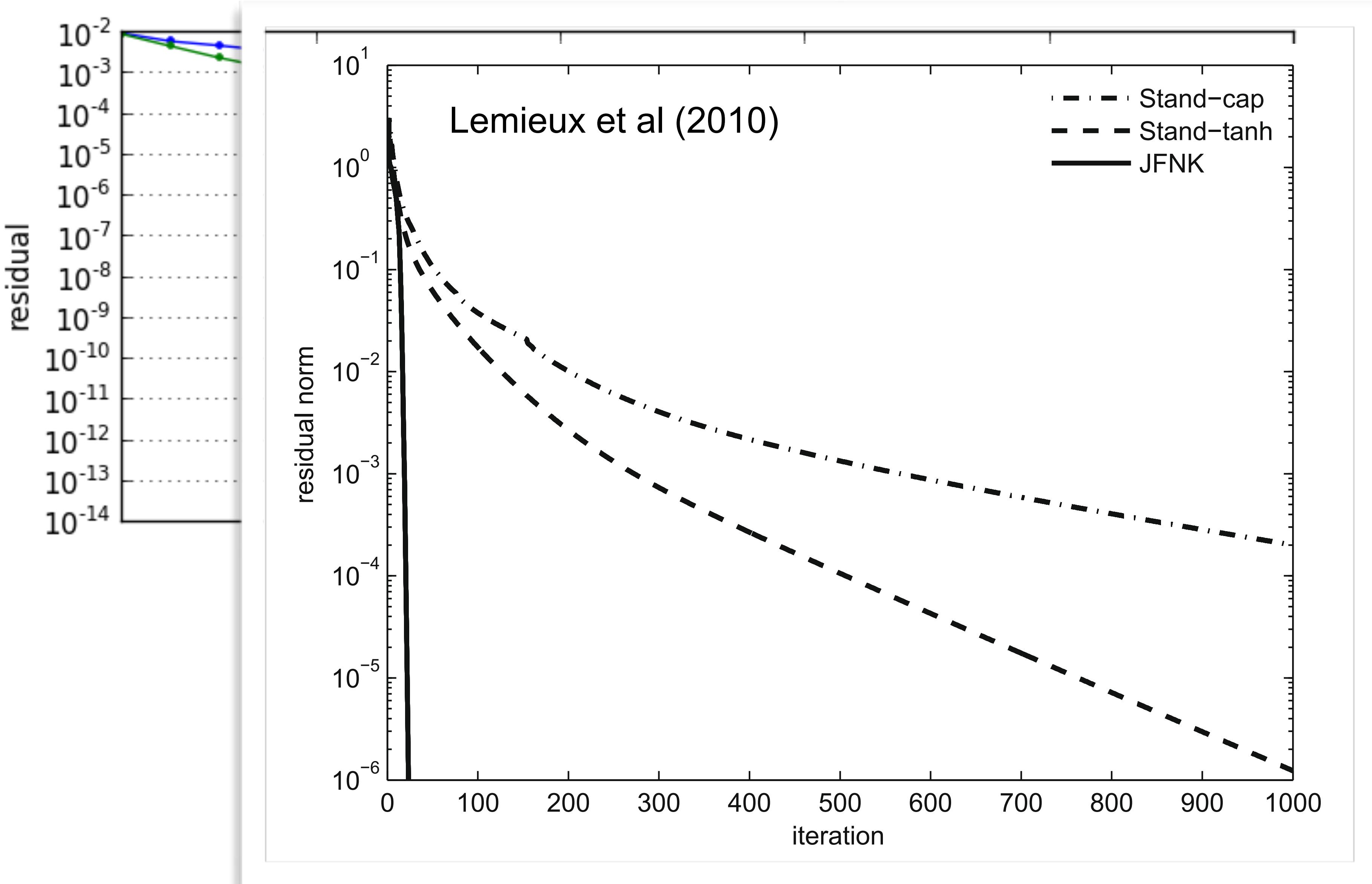
$\Rightarrow$ solve $\quad \mathbf{F}'_{n-1}\,\delta\mathbf{u} = -\,\mathbf{F}(\mathbf{u}_{n-1}) \quad \Rightarrow \quad \mathbf{u}_n = \mathbf{u}_{n-1} + \delta\mathbf{u}$

- better (quadratic) convergence near minimum (Lemieux et al. 2010, 2012, Losch et al 2014)
- preconditioner for Krylov solver necessary
- expensive
- unstable, especially at high resolution
- stabilization (e.g. Mehlmann and Richter 2017, involves mixing JFNK and Picard methods)

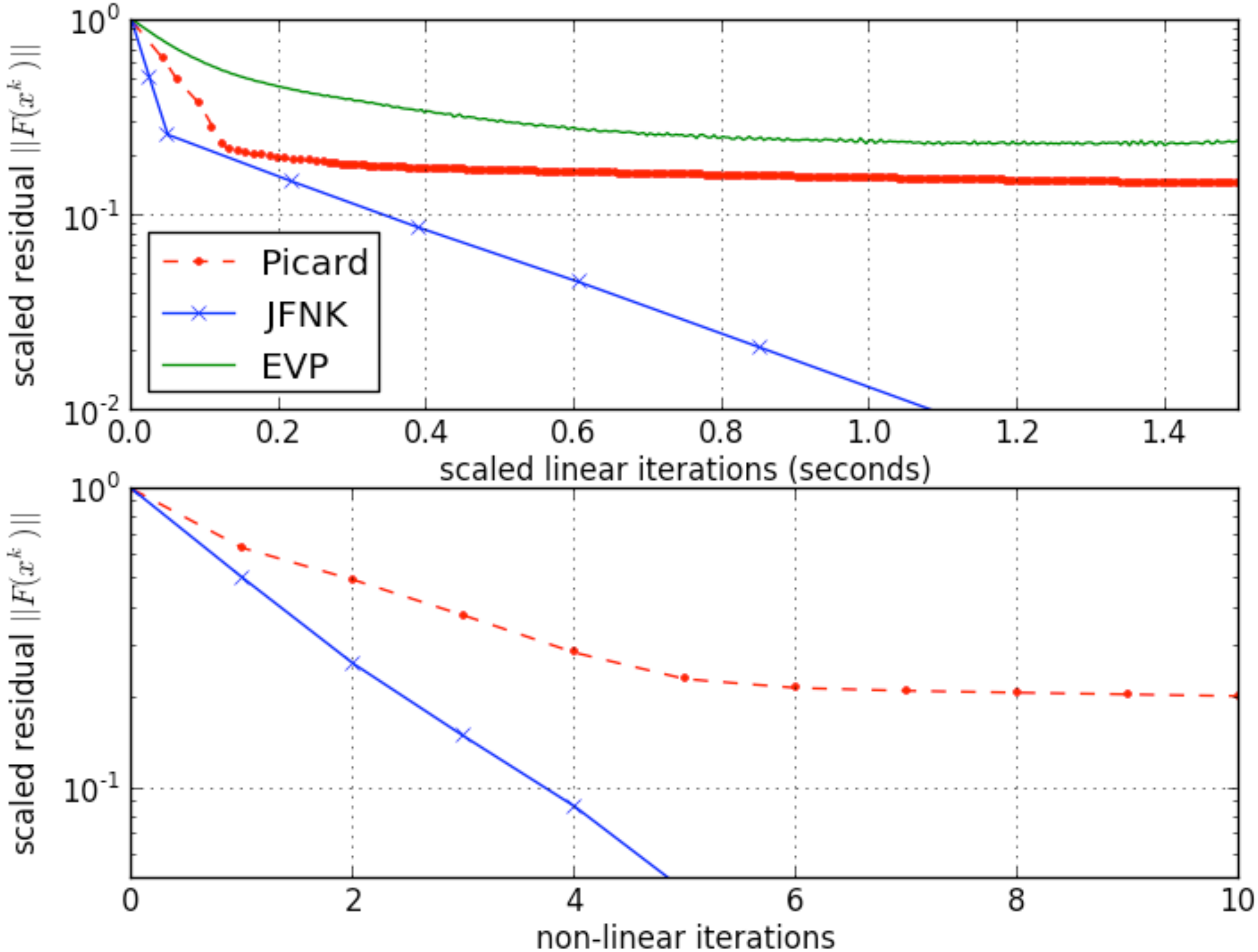
Picard vs. JFNK



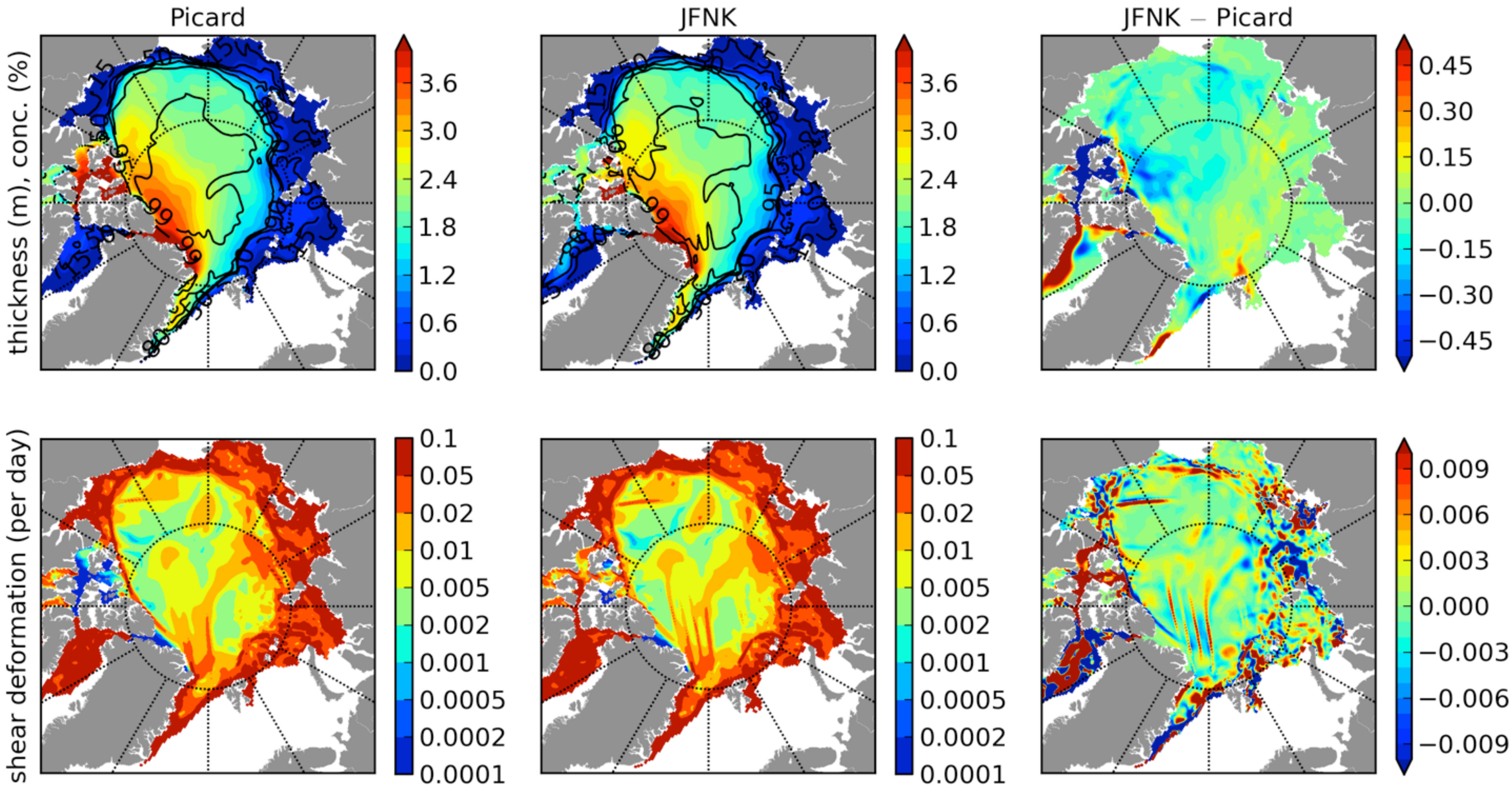
Picard vs. JFNK



“Timing” of solvers



Does it matter?



Losch et al. (2014)

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solution method: EVP variants

$$\sigma_{ij} = \frac{P}{2\Delta}\left\{2\dot{\epsilon}_{ij}e^{-2} + \left[(1-e^{-2})(\dot{\epsilon}_{11}+\dot{\epsilon}_{22}) - \Delta\right]\delta_{ij}\right\}$$

$$\Leftrightarrow \quad \frac{\Delta e^2}{P}\sigma_{ij} + \left[\frac{\Delta(1-e^2)}{2P}(\sigma_{11}+\sigma_{22}) + \frac{\Delta}{2}\right]\delta_{ij} = \dot{\epsilon}_{ij}$$

- Hunke and Dukowicz (1997)
- does not converge (definitely not to VP, Lemieux et al. 2012, Losch and Danilov 2012)
- adding inertial term to momentum equations fixes convergence (Lemieux et al. 2012, Bouillon et al 2013)
- m(odified)EVP, a(daptive)EVP (Kimmritz et al 2015, 2016, 2017)

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$$\Leftrightarrow \left(\frac{1}{E}\frac{\partial\sigma_{ij}}{\partial t} + \right)\frac{\Delta e^2}{P}\sigma_{ij} + \left[\frac{\Delta(1-e^2)}{2P}(\sigma_{11}+\sigma_{22}) + \frac{\Delta}{2}\right]\delta_{ij} = \dot{\epsilon}_{ij}$$

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New EVP equations

based on Lemieux et al. (2012), Bouillon et al. (2013), add “inertial-like” term to momentum equations

$$\boldsymbol{\sigma}^{p+1} - \boldsymbol{\sigma}^{p} = \frac{1}{\alpha}\Big(\boldsymbol{\sigma}(\mathbf{u}^{p}) - \boldsymbol{\sigma}^{p}\Big),$$

$$\mathbf{u}^{p+1} - \mathbf{u}^{p} = \frac{1}{\beta}\Big(\frac{\Delta t}{m}\nabla\cdot\boldsymbol{\sigma}^{p+1} + \frac{\Delta t}{m}\mathbf{R}^{p+1/2} \qquad\qquad \Big)$$

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now, with $\sigma^{p+1} = \lim_{p\to\infty} \sigma^{p}$ and $\mathbf{u}_{n+1} := \lim_{p\to\infty} \mathbf{u}^{p}$

the discretized equations converge to true VP

$$\frac{m}{\Delta t}\Big(\mathbf{u}_{n+1} - \mathbf{u}_n\Big) = \nabla\cdot\boldsymbol{\sigma}(\mathbf{u}_{n+1}) + \mathbf{R}^{*}$$

with $\mathbf{R}^{*} := \lim_{p\to\infty} \mathbf{R}^{p+1/2}$

New EVP equations

New momentum equations

$$\boldsymbol{\sigma}^{p+1} - \boldsymbol{\sigma}^{p} = \frac{1}{\alpha}\Big(\boldsymbol{\sigma}(\mathbf{u}^{p}) - \boldsymbol{\sigma}^{p}\Big),$$

$$\mathbf{u}^{p+1} - \mathbf{u}^{p} = \frac{1}{\beta}\Big(\frac{\Delta t}{m}\nabla \cdot \boldsymbol{\sigma}^{p+1} + \frac{\Delta t}{m}\mathbf{R}^{p+1/2} \boxed{+ \mathbf{u}_{n} - \mathbf{u}^{p}}\Big)$$

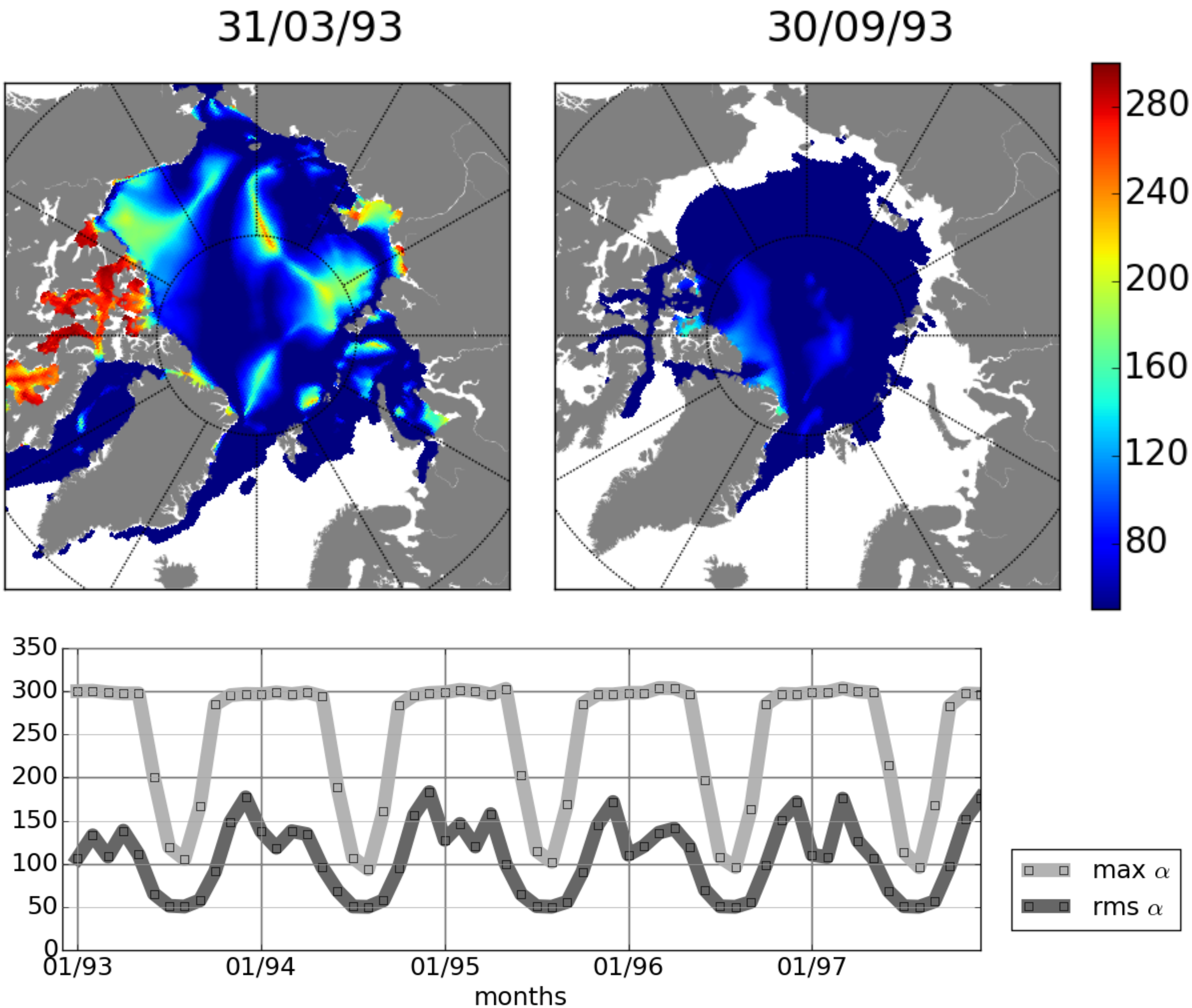
with $$\alpha\beta \gg \gamma = \frac{P}{2\Delta}\frac{(c\pi)^2}{A}\frac{\Delta t}{m}$$

from stability analysis (Kimmritz et al, 2015, 2016).

modified EVP: α, β = constant, order(300)

adaptive EVP: $\alpha = \beta = (4\gamma)^{1/2}$

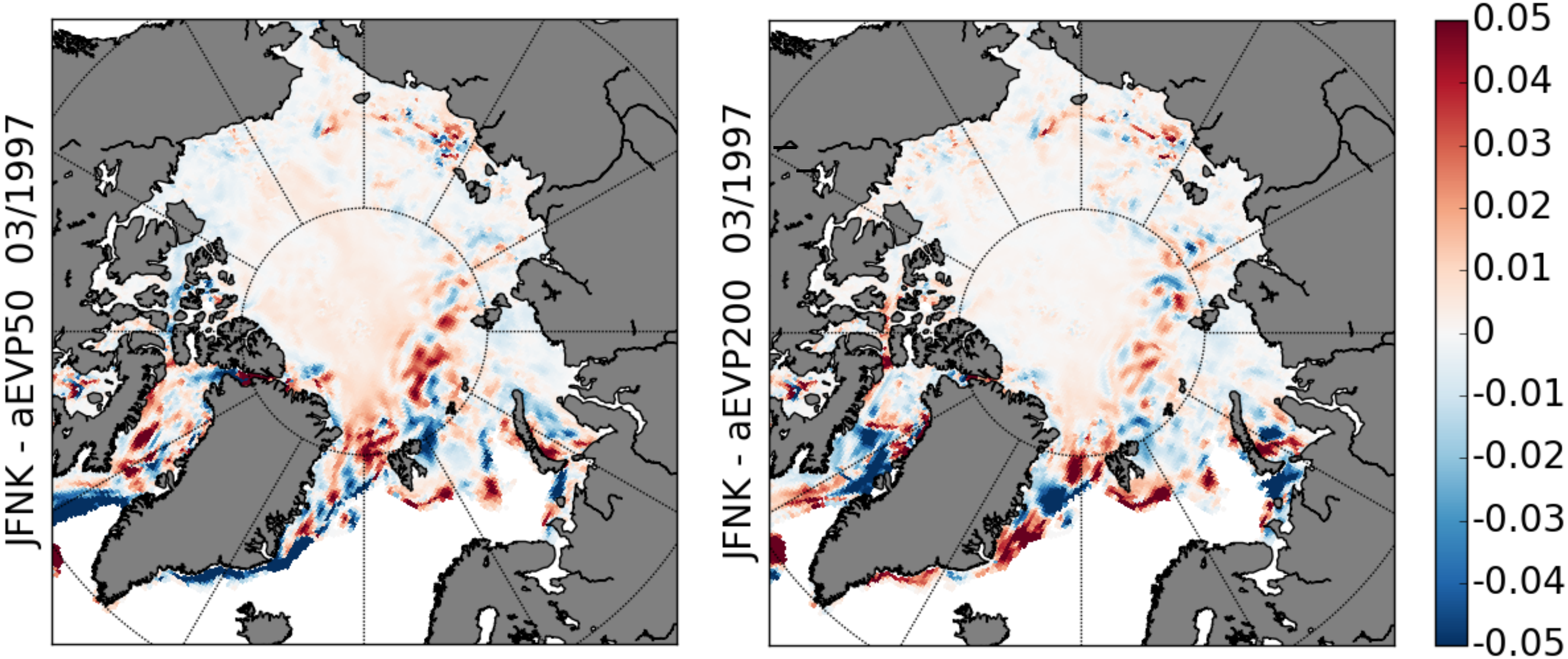
Parameter α (aEVP, N = 500)



Kimmritz, Losch, Danilov (2017)

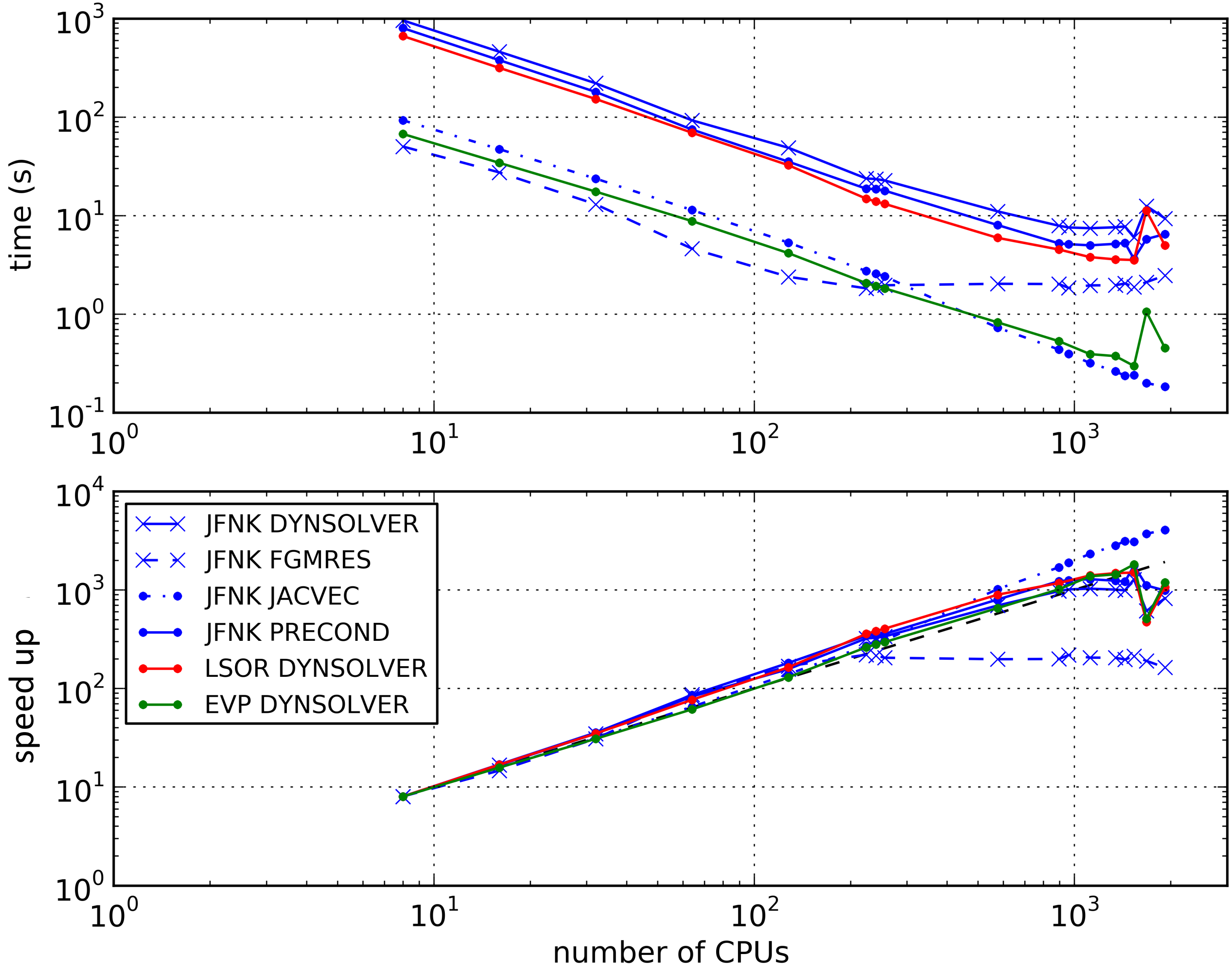
Convergence to VP solution: JFNK — aEVP difference in ice thickness (m) at 27 km resolution

50 EVP iterations 200 EVP iterations



Kimmritz et al. (2017)

MITgcm as a testbed: scalability



Losch et al. (2014)

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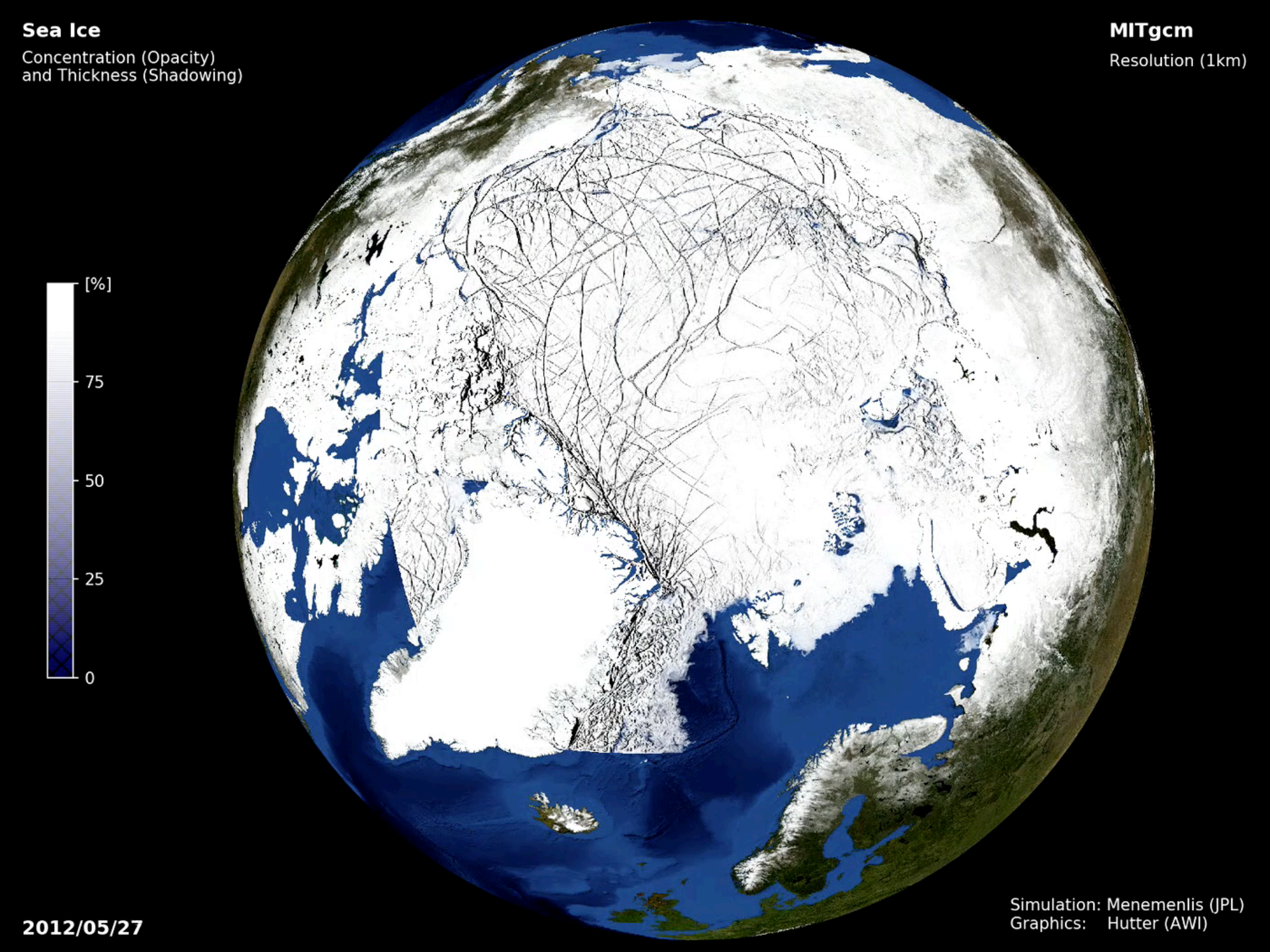
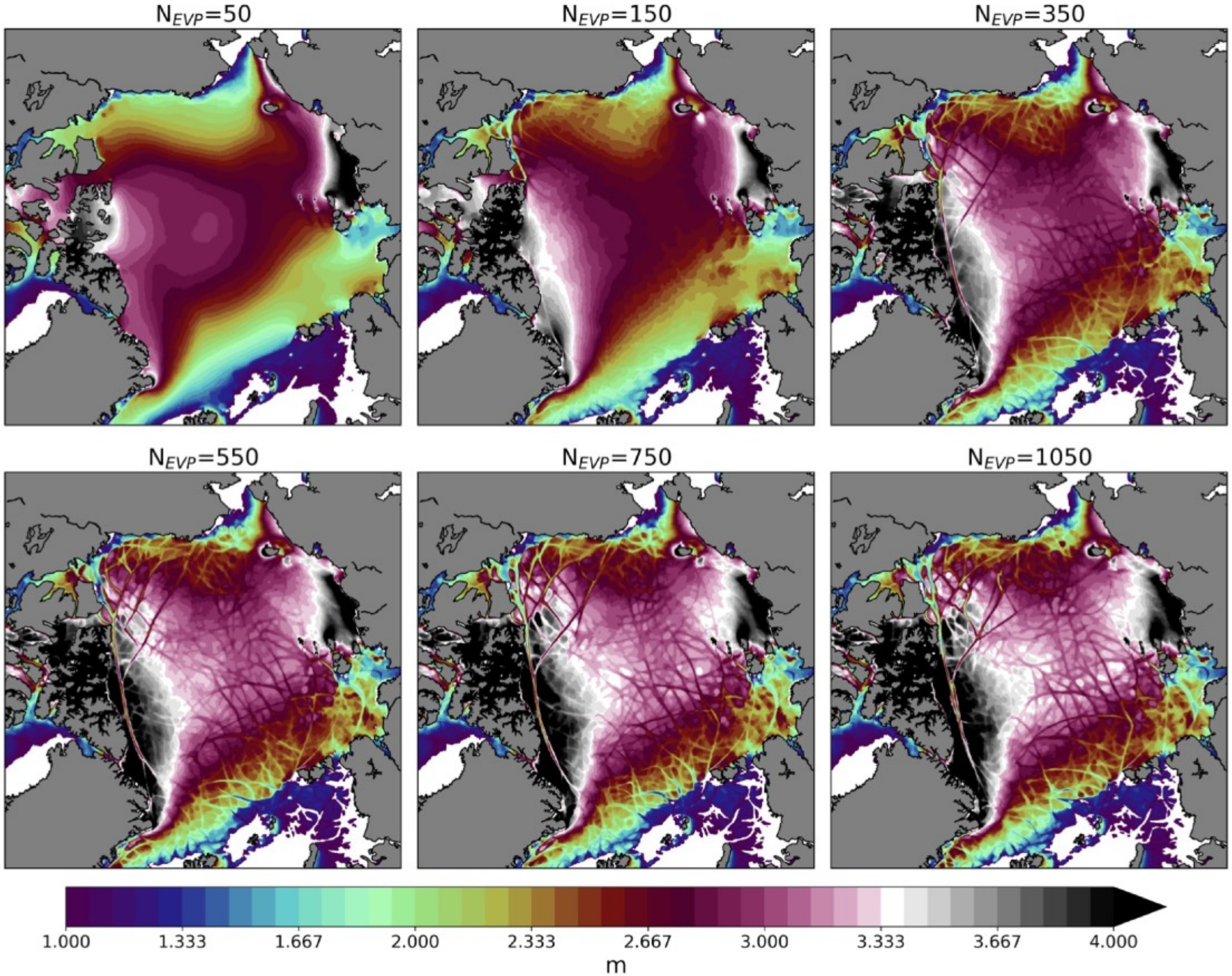
high resolution simulations


Sea Ice
Concentration (Opacity)
and Thickness (Shadowing)
MITgcm
Resolution (1km)
[%]
75
50
25
0
2012/05/27
Simulation: Menemenlis (JPL)
Graphics: Hutter (AWI)

EVP "convergence" (in FESOM)



Koldunov et al., submitted manuscript, grid resolution ~ 4 km

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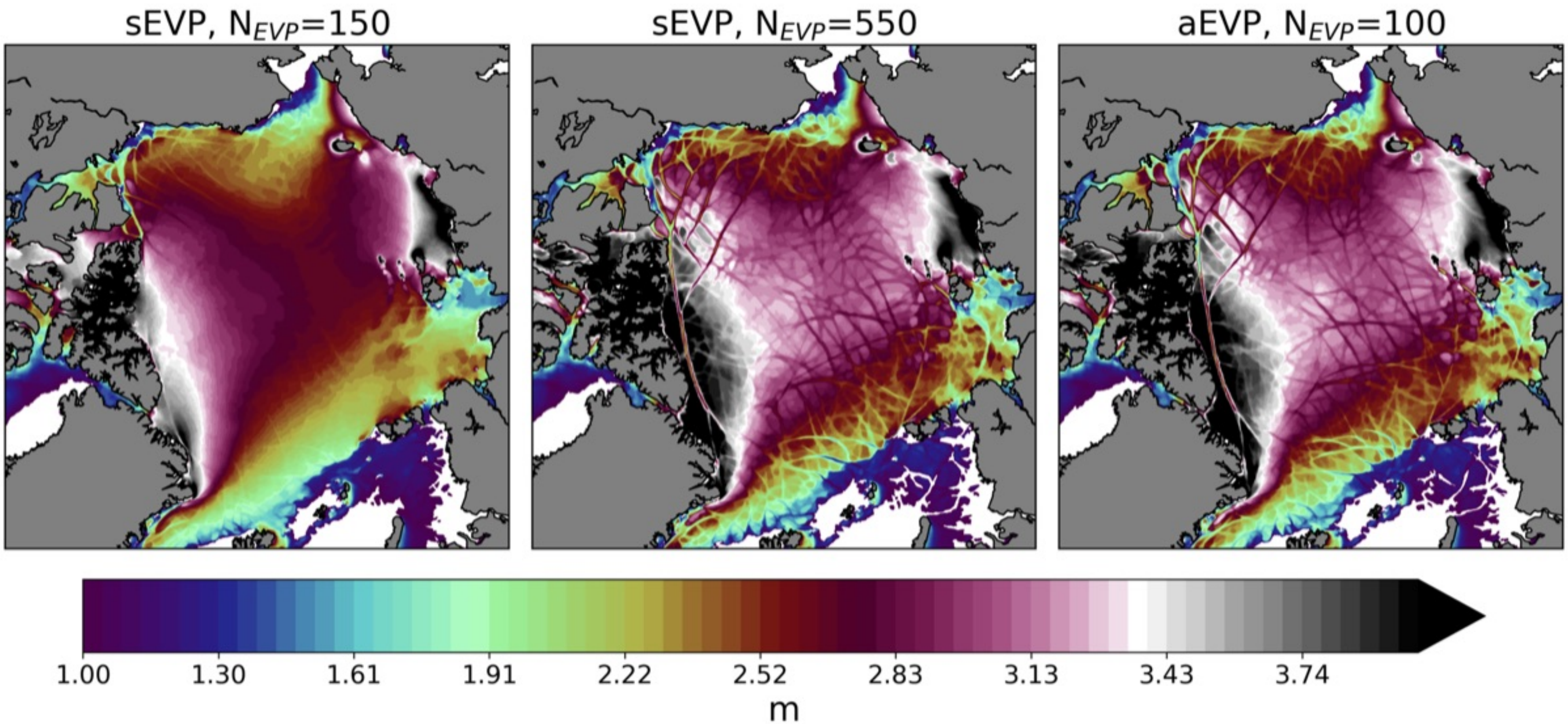
EVP “convergence” (in FESOM)



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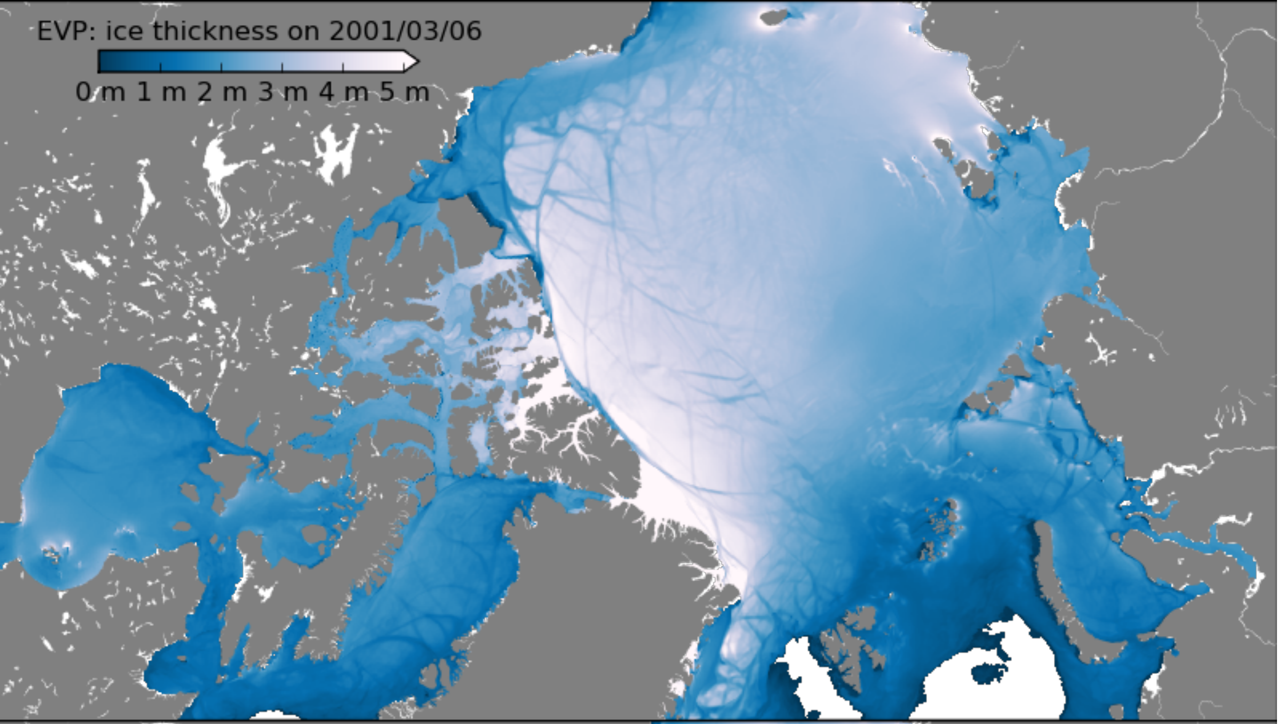
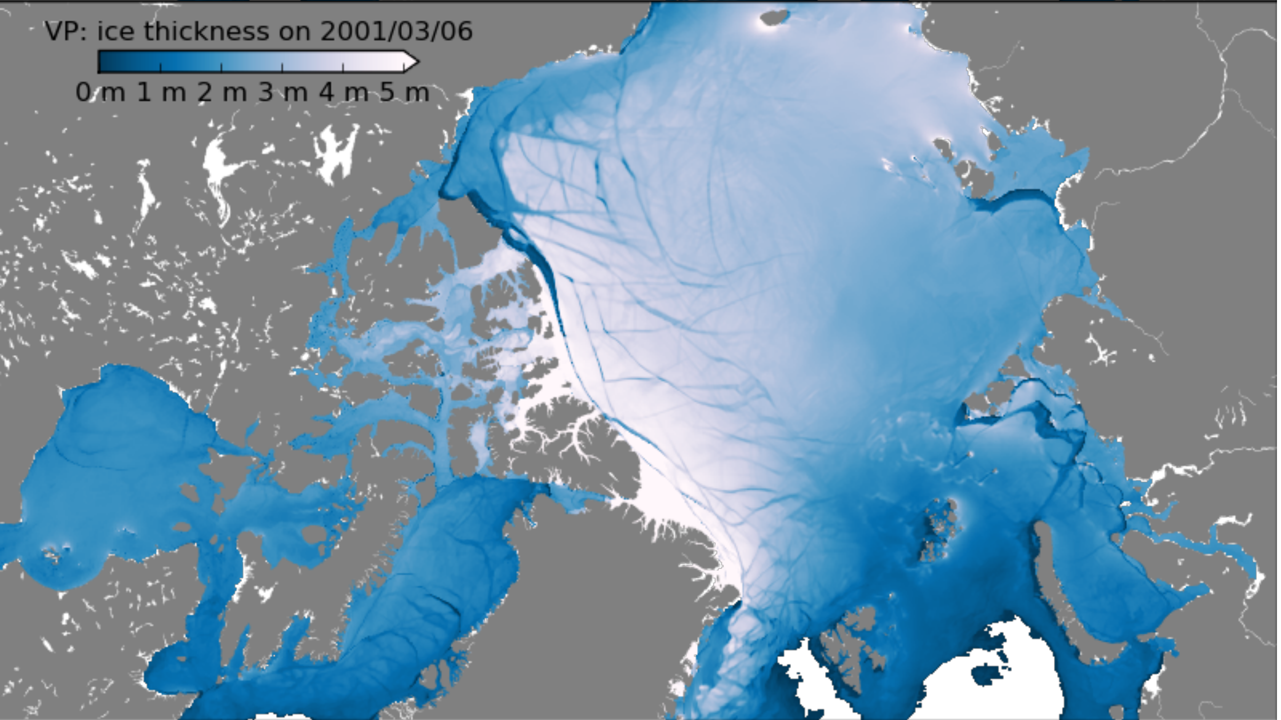




Convergence to VP solution:

ice thickness (m) at 4.5 km grid spacing

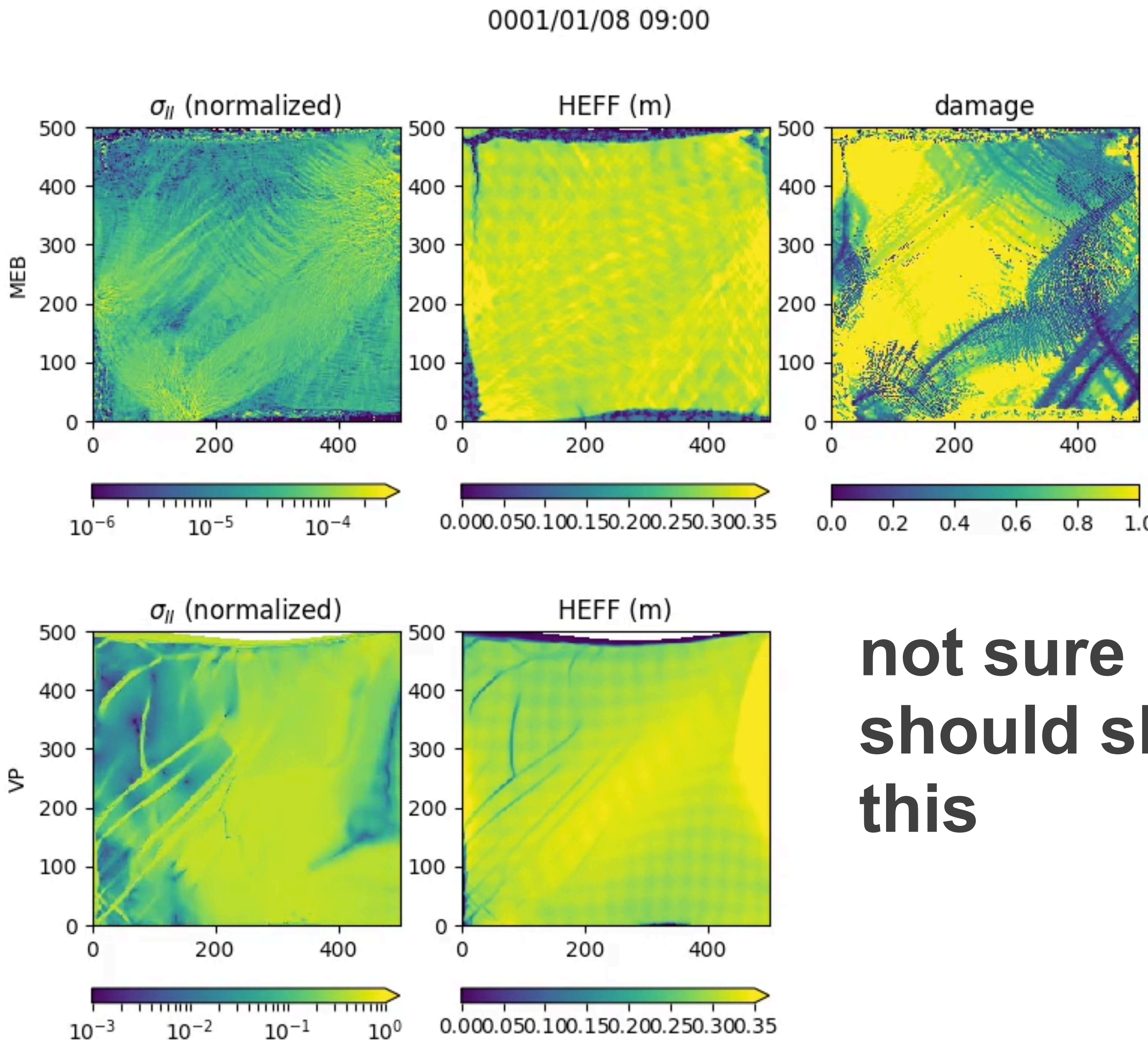
$$\alpha\beta \gg \gamma = \zeta \frac{(c\pi)^2}{A_c} \frac{\Delta t}{m}$$

stability parameter depends on grid spacing and local ice viscosity

Maxwell Elasto-Brittle rheology

$$\frac{1}{E}\frac{\partial \sigma}{\partial t} + \frac{1}{\lambda}\sigma = K : \dot{\epsilon}$$

- violation of a Mohr-Coulomb failure criterion determines a damage parameter
- damage parameter affects ice strength, elasticity
- (Girard et al 2011, Rampal et al 2015, Dansereau et al 2016)
- In the pipeline for the MITgcm



not sure if I should show this

Summary

- Viscous Plastic rheology:
  - Picard solvers with LSR and Krylov solvers
  - JFNK solver
  - many EVP variants, especially stable EVP algorithms (Kimmritz et al. 2015, 2016)
- Maxwell Elasto-Brittle rheology (not quite, yet)
- all in the same code framework (no confounders in comparisons)
- main issues remain, especially at high resolution: convergence, stability vs. geophysical plausibility vs. time to solution