Plume spreading test case for coastal models

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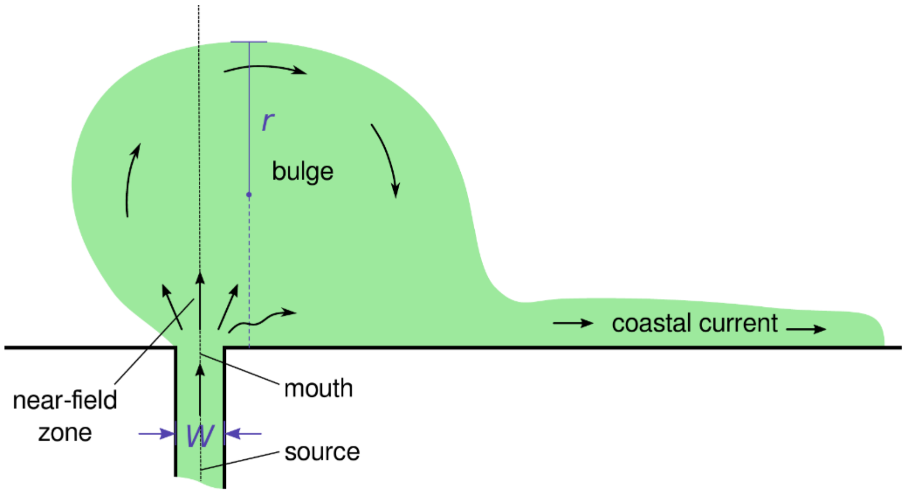
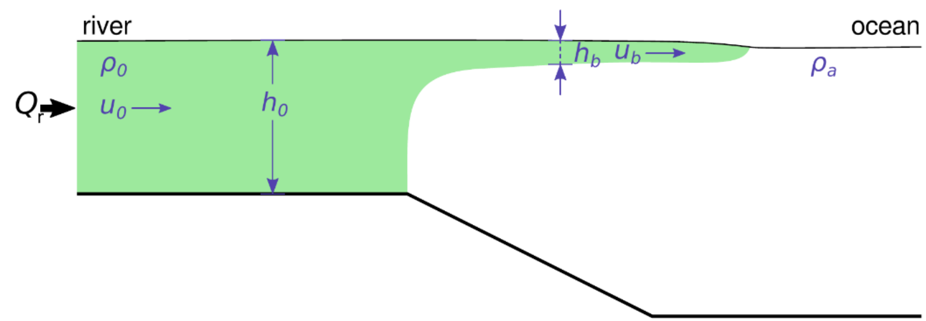
Introduction





Estuary-shelf system whose dynamics combine nonlinear flow regimes with sharp frontal boundaries and linear regimes with cross-shore geostrophic balance.

Plume simulations is an excellent benchmark for the ocean-model intercomparison.

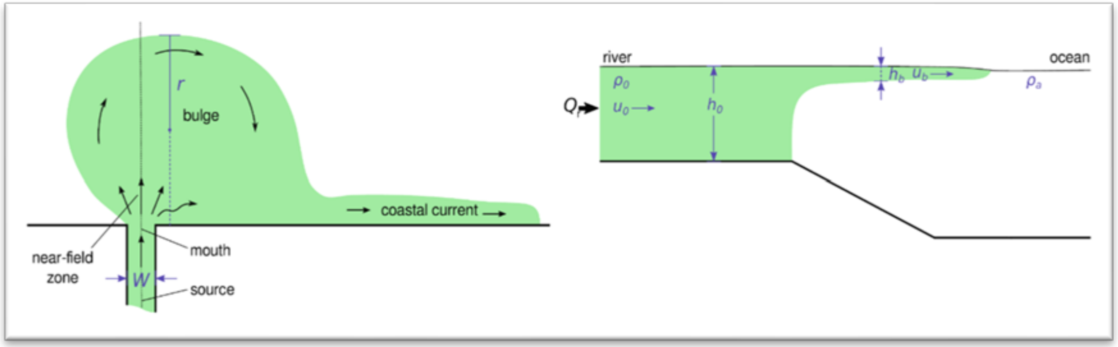
Plume test case

Let‘s create estuary-shelf system:

- behavior of which can be predicted analytically;
- is highly sensitive to physical or numerical dissipation and mixing;
- allows transparent diagnostic of the level of the numerical mixing and general performance of the model;
- suitable for the diferent model types (discretization, structured/unstructured grid based and etc.).

Parametres to play with:

- river discharge rate;
- width of the estuary;
- bathymetry;
- density gradient;
- Coriolis parameter.



How should look like a final system at first glance:

- thin surface-advected plume
- all the classical zones present
- pronounced bulge (75 % of the river discharge should stay there)
- ensuring supercritical flow in the river mouth area (long internal wave disturbances can travel only upstream);
- river channel oriented perpendicularly to the shelf.

Analytical prediction of plume behaviour

What?

- position of the lift-off zone;
- bulge characteristics at a given time;
- depth of the coastal current, its cross-front width, and velocity.

How?

- Assuming flow in cyclostropic balance; using Bernoulli function for buoyant layer (see some details in Yankovsky & Chapman articles) ← Bulge
Mouth area
- Based on analysis of two-layer exchange flow through contractions with a barotropic component treated by Armi & Farmer; ← Mouth area
- Using laboratori studies (mainly done by Avicola and Huq); ← Bulge
- Using series of classifications and analysis based on Froude number, baroclinic Rossby radius and estuary geometry (see works by Garvine, Horner-Devine and Hetland). ← Bulge
Coastal current

The system is treated as a two-layer one for analytical consideration.

The diffusive and viscouse processes are not considered!

Final plume characteristics

$W = 0.5$ km is the width of the mouth,

$h_0 = 10$ m is the inflow depth,

$Q_r = 3000(3900)$ m^3/s is the river discharge rate,

$f = 1.2 \cdot 10^{-4}$ s^{-1} is the Coriolis parameter,

$u_0 \cong 0.6(0.78)$ m/s $(Q_r/(Wh_0))$ is the river velocity in the channel in a steady regime,

$\rho_r = 1000.65$ kg/m^3 is the density of river water,

$\rho_0 = 1023.66$ kg/m^3 is the ambient/shelf water density,

$g' = g\frac{\rho_a - \rho_0}{\rho_a} \approx 0.225$ m/s^2 is the reduced gravity,

$c_0 = \sqrt{g'h_0} \approx 1.5$ m/s is the reference phase speed,

$r_0 = \frac{c_0}{f} = 12.5$ km is the inflow Rossby radius,

$L_0 = \frac{u_0}{f} = 5(6.5)$ km is the inertial radius,

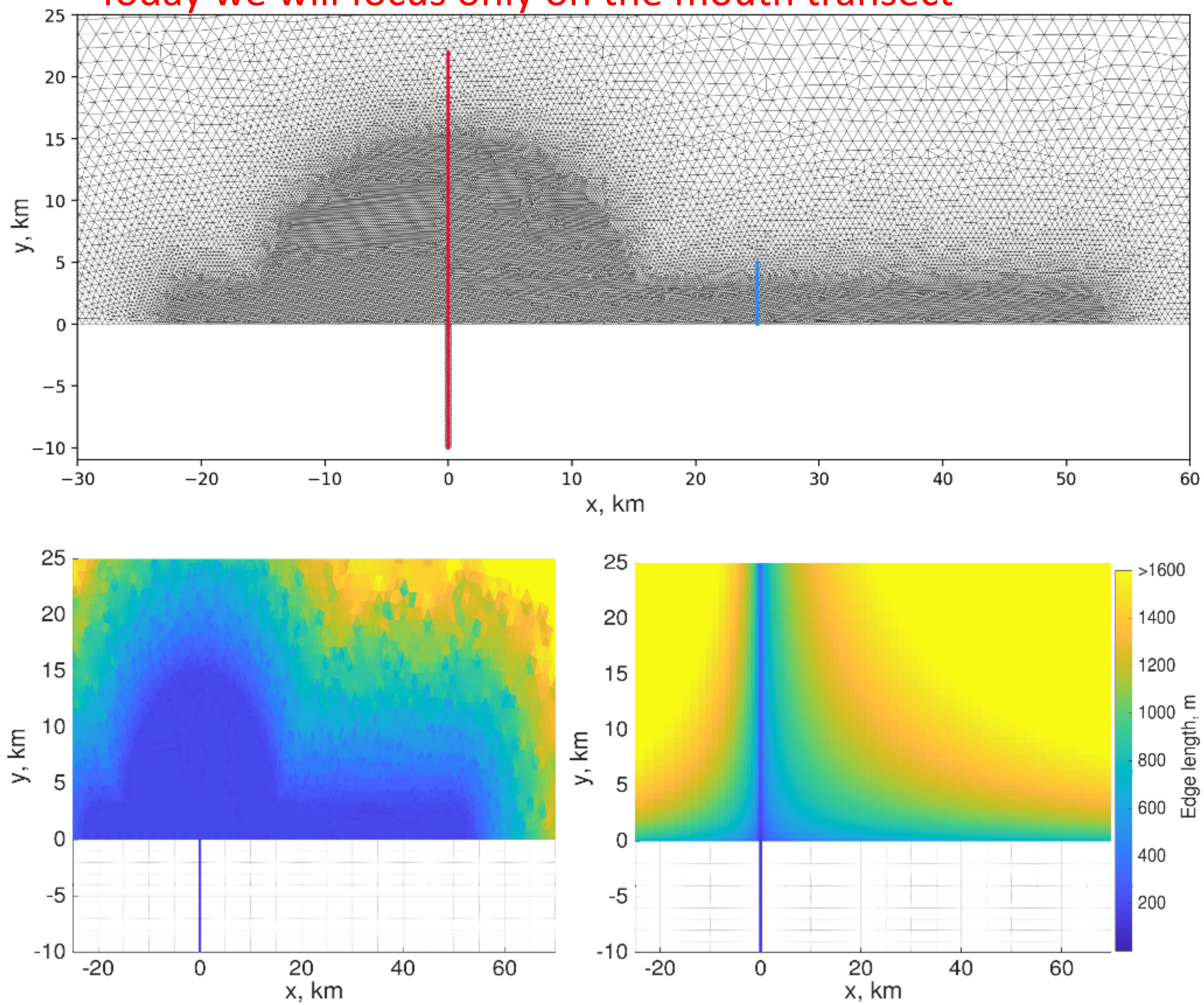
$L_b = \sqrt[4]{\frac{2Q_r g'}{f^3}} \approx 5.28(5.65)$ km is the internal Rossby radius for the bulge based on the geostrophic depth,

$Fr_0 = \frac{u_0}{c_0} = 0.4(0.52)$ is the initial Froude number,

$T_0 = \frac{2\pi}{f} = 14.54$ h is the inertial (rotational) period.

Grids/transects

Today we will focus only on the mouth transect



Setup details

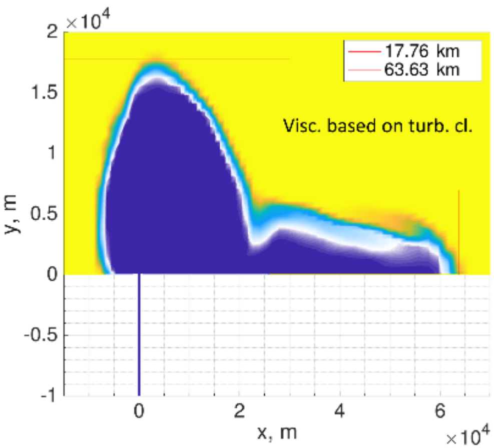
- Vertical resolution: 20-40 sigma layers
- Slope only on the shelf (0.003); depth range: 10..30
- Bottom friction is turned off
- k-e turbulence scheme (coefficients from Canuto et al. (2001) (version A))
- Salinity range: 0 ... 30
- Linear equation of state, temperature is CONST
- No open boundaries
- No atmospheric and no tidal forcing

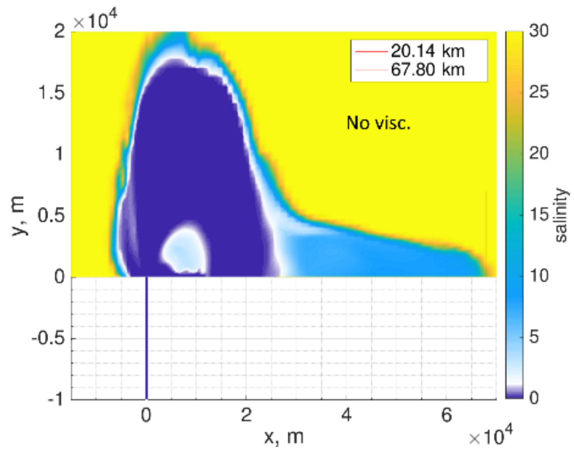
Setup details: eddy viscosity, eddy diffusivity

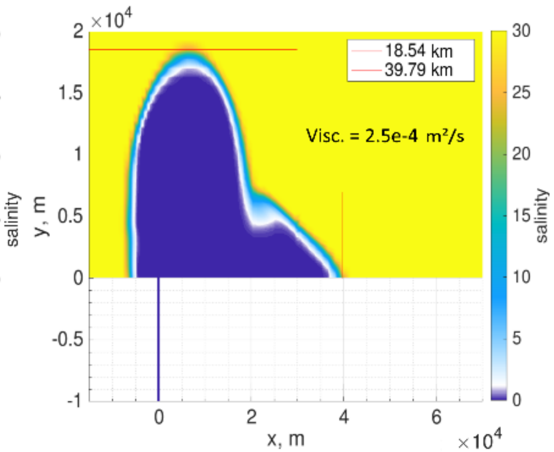
Eddy diffusivity is off!

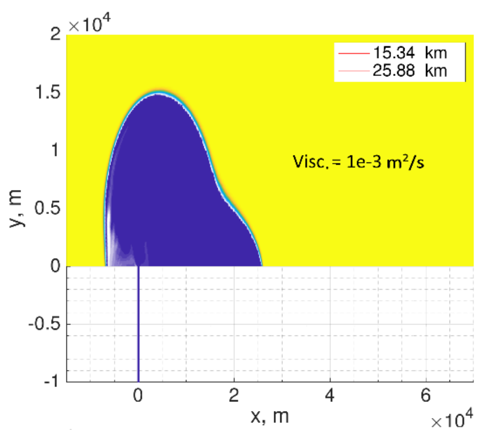
All runs listed there were carried out with the same turbulence closure for vertical viscosity.

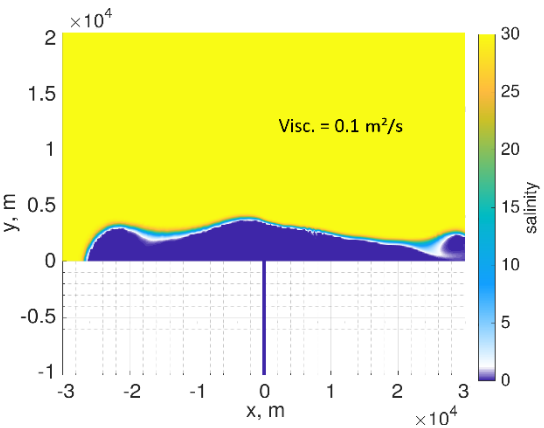
Eddy viscosity?











Eddy viscosity stays!

The reason is the presence of numerical mixing in the system, which dilutes the thin freshwater layer, and other numerical inaccuracies, which have in this case pronounced consequences.

Sources of numerical mixing

Spurious numerical mixing can be attributed to the:

- advection schemes;
- vertical grid;
- discretization;
- time-stepping.

Effect of spurious mixing on plume dynamics is still poorly understood!

Models & Advection Schemas

FESOM-C

upwind (2d order in space and in time), MIURA (2d order in space and time), MUSCL (combination of the 3d order and 4th order fluxes in space, 2d order in time)
+ fct limiters (three options to constrain the antidiffusive flux which is added to the solution obtained with the positivity-preserving low-order scheme)
All 4th order vertically

THETIS

upwind (2d order in space and in time) + geometric slope limiter (if tracer value at a node exceeds the maximum mean value of the neighboring elements, it is marked as an overshoot)

GETM

directional splitting with TVD scheme of the 2d order in space and time **+** superbee limiter , which is known by its anti-diffusive (anti-viscous) character
HSIMT TVD scheme (3d order in space and 2d order temporally) + Sweby's limiter

Surface salinity

High order hybrid advection scheme

2d order upwind

Mouth area

THETIS runs with upwind tracer advection scheme

With physical diffusion

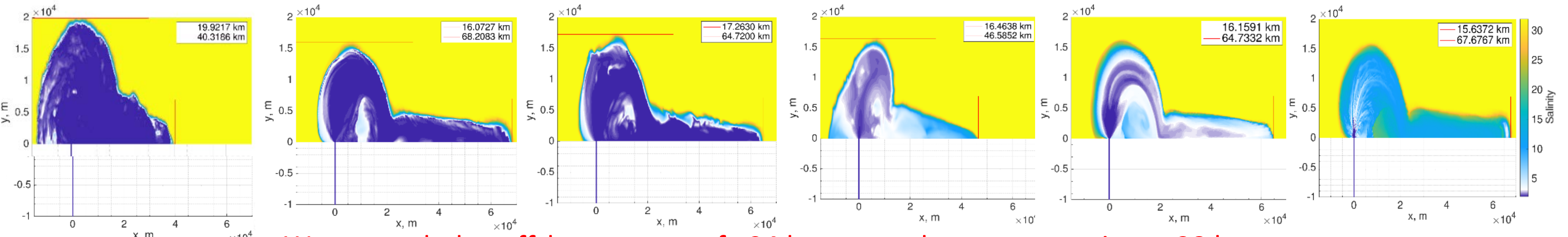
No physical diffusion

Let's increase discharge from 3000 m^3/s to 3900 m^3/s. Based on the analysis of two-layer exchange flow through contractions with a barotropic component treated by Armi & Farmer no penetration should happen!

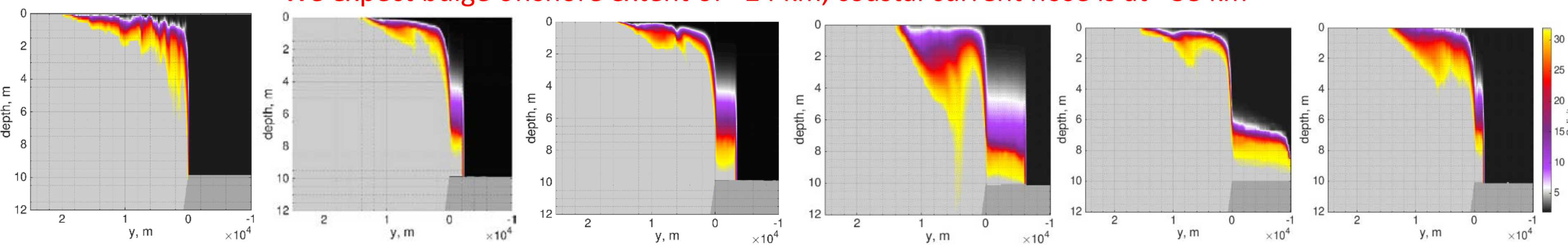
Numerical diffusion attributed to the advection scheme provokes penetration;

Relatively high viscosity blocks penetration of the dense water into the river channel.

Intercomparison based on advection scheme



We expect bulge offshore extent of ~24 km, coastal current nose is at ~38 km



(3-4th) muscl type +fct1

(3-4th) muscl type +fct3

(2d) MIURA +fct1

(2d) upwind +no lim.

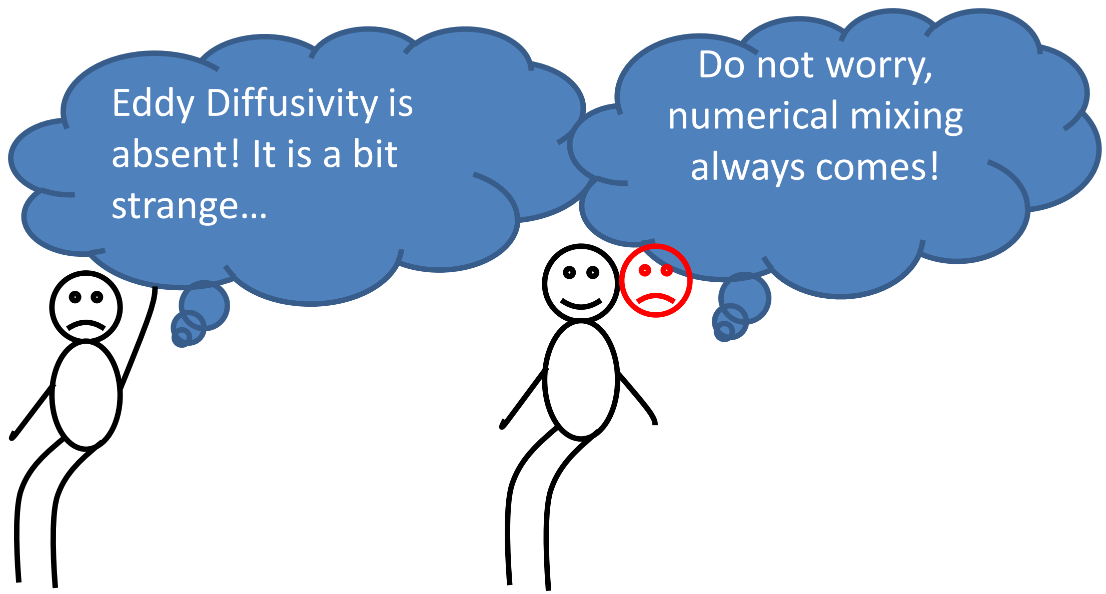
(2d) upwind +geom. lim

(3-4th) muscl type +fct1, with phys. Diff.

- If numerical mixing is larger, we get a smaller bulge offshore extent, a blurry surface layer, a thicker plume, a bulge center closer to the coast, and a larger coastal current discharge;
- Accuracy in reproducing the analytical solution depends less on the type of model discretization or computational grid than it does on the type of advection scheme;
- The order of accuracy of a given scheme is as important as the type of limiter.

Numerical mixing

Now we have qualitative estimates about effect of numerical mixing on plume dynamics….



Can we quantify?

How to diagnose numerical mixing?

The balance equation for salinity + the mass conservation law

Budget equation for the salt content integrated over all salinities between the river salinity S_r and the salinity of the isohaline, S:

diffusive salinity flux

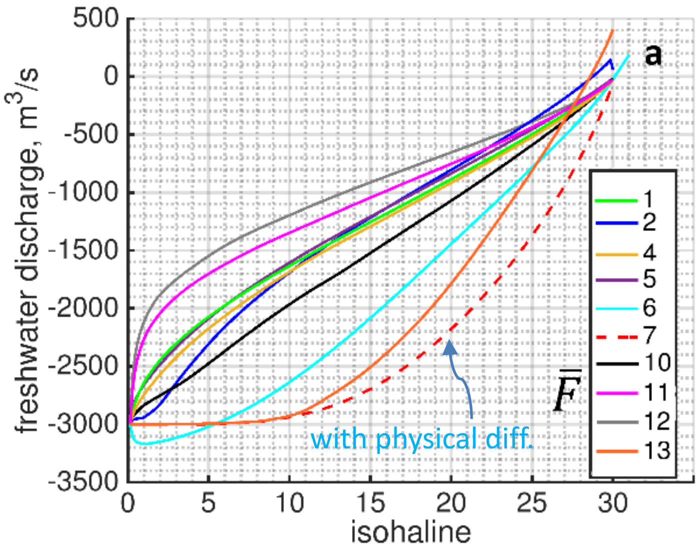
No physical diffusion in the system, zero river salinity

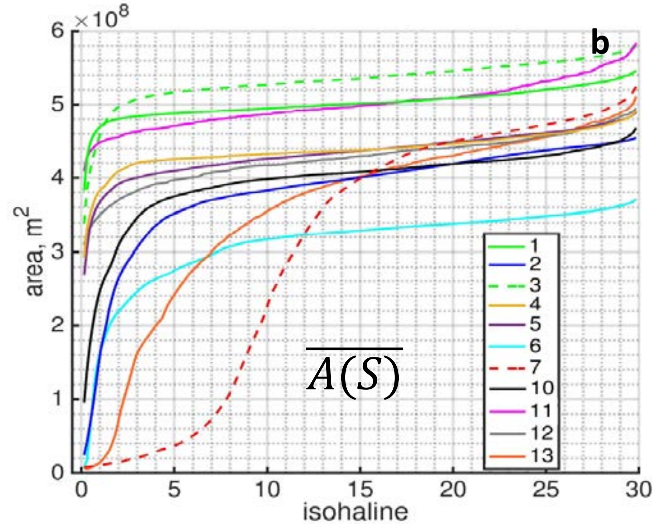
total freshwater transport across isohaline due to numerical mixing

We can quantify level of the numerical mixing in the system

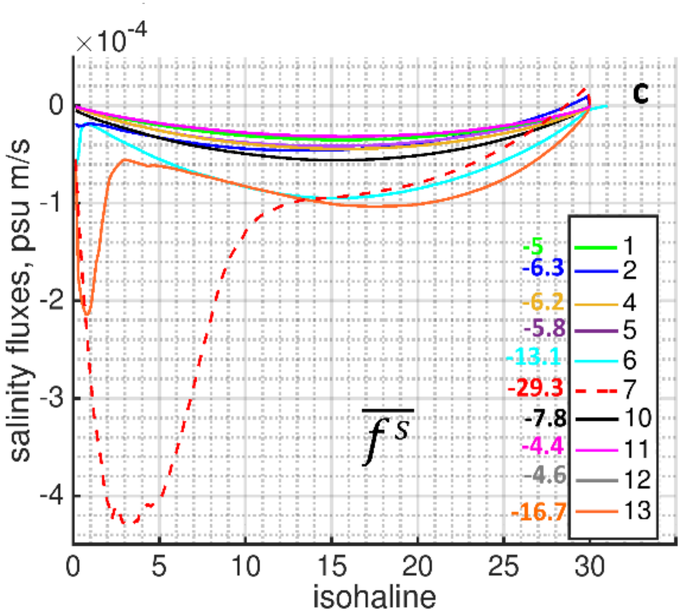
Note, numerical mixing may also include antidiffusive effects!

Numerical mixing





The bulge spreading characterizes largely the horizontal part of dynamics, whereas total salinity fluxes characterize the vertical.



The total salinity fluxes characterize the net level of numerical diffusion in the system, or how closely the system adheres to the expected two-layer system with only two salinity classes.

The smaller the absolute value of salinity fluxes, the lower the level of the numerical mixing in the considered simulation.

Take home messages

- All models reproduce the four prototypical zones, preserve freshwater volume in the system, and are stable.
- If numerical mixing is larger, we get a smaller bulge offshore extent, a thicker plume, a bulge center closer to the coast, and often a larger coastal current discharge;
- The order of accuracy of a given scheme is as important as the type of limiter;
- The fct1, fct2, geometrical and Superbee limiters outperformed fct3 and Sweby's limiters;
- Among the considered advection schemes, the best performer was a hybrid MUSCL-type advection scheme (3d-4th order) combined with the fct1 limiter.

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26 Feb 2021

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Additional slides

FESOM-C

- Cell-vertex finite volume discretization
- Share part of the infrastructure with FESOM
- Any configurations of triangular, quadrangular or hybrid meshes
- External/internal modes
- Terrain following vertical coordinate
- 3rd-order upwind horizontal advection schemes
- Implicit 3d-order vertical advection schemes, implicit vertical viscosity
- Biharmonic horizontal viscosity augmented to the Smagorinsky viscosity
- GOTM turbulence library for the vertical mixing
- Rivers through solid boundary in streaming form/ Rivers as open boundary conditions
- Tidal potential /Open boundary prescription of amplitudes and phases for 12 harmonics
- Wetting/drying
- Simple sediment module (one type of the noncohesive sediments)

GETM

- C-staggered structured finite-volume grid
- Vertically boundary-following coordinates with adaptive internal layer distribution
- Cartesian, spherical and curvilinear horizontal coordinates
- Nonlinear free surface with split-explicit mode-splitting and robust drying-and-flooding capability
- State-of-the-art vertical turbulence closure from GOTM
- Efficient subdomain decomposition for massively parallel applications

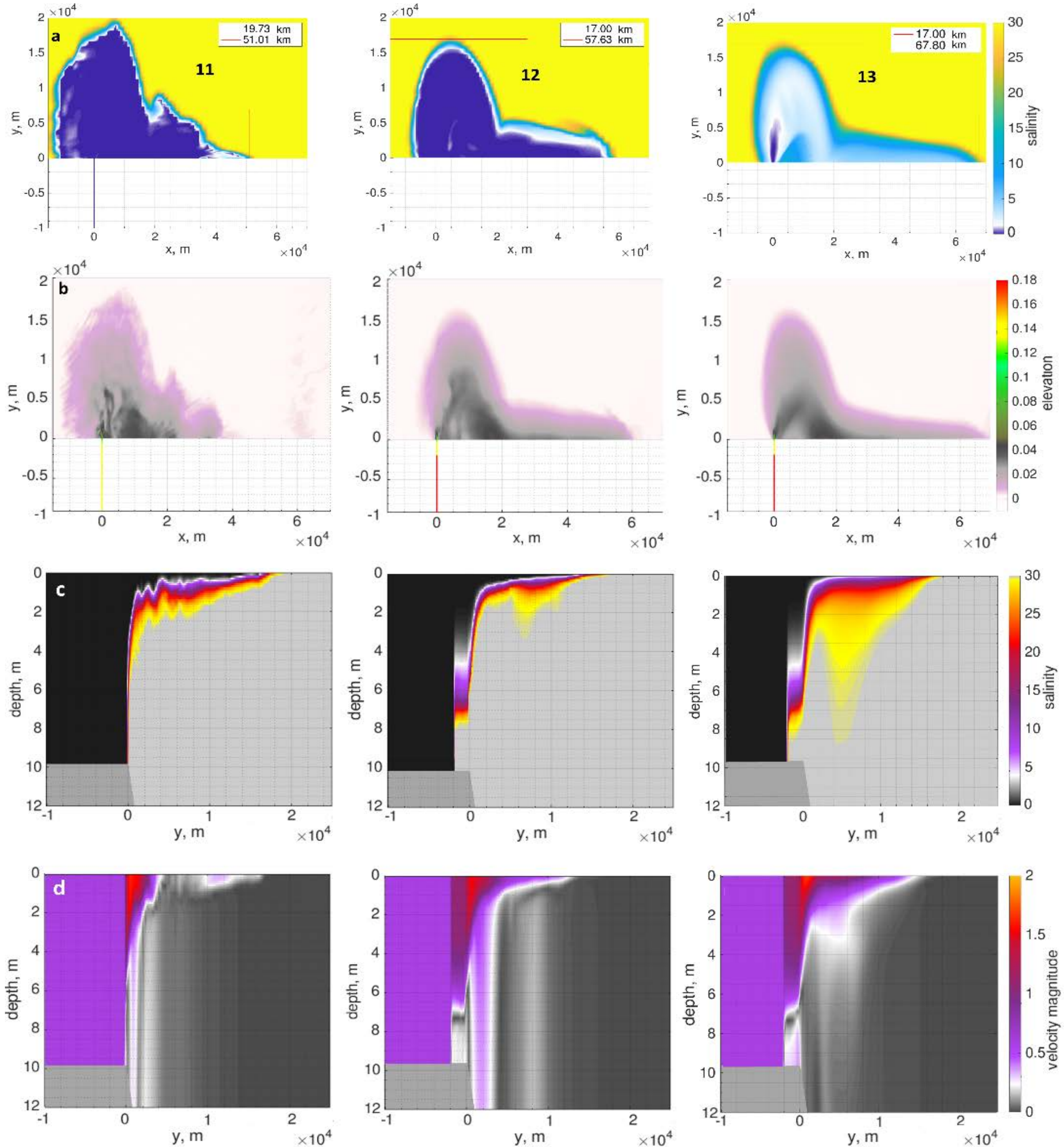
THETIS

- Finite Element formulation for 3D hydrostatic equations
- Unstructured horizontal mesh: triangles or quads
- Discontinuous Galerkin discretization (p=1)
- 2nd order split-implicit time integration
- Upwind 2d order adv. Scheme with slope limiter
- Arbitrary-Largrangian-Eulerian vertical grid
- Native implementation of GLS turbulence closure model
- Implemented in Python on Firedrake FE framework
- Uses Domain Specific Language (DLS) and automated code generation

http://thetisproject.org

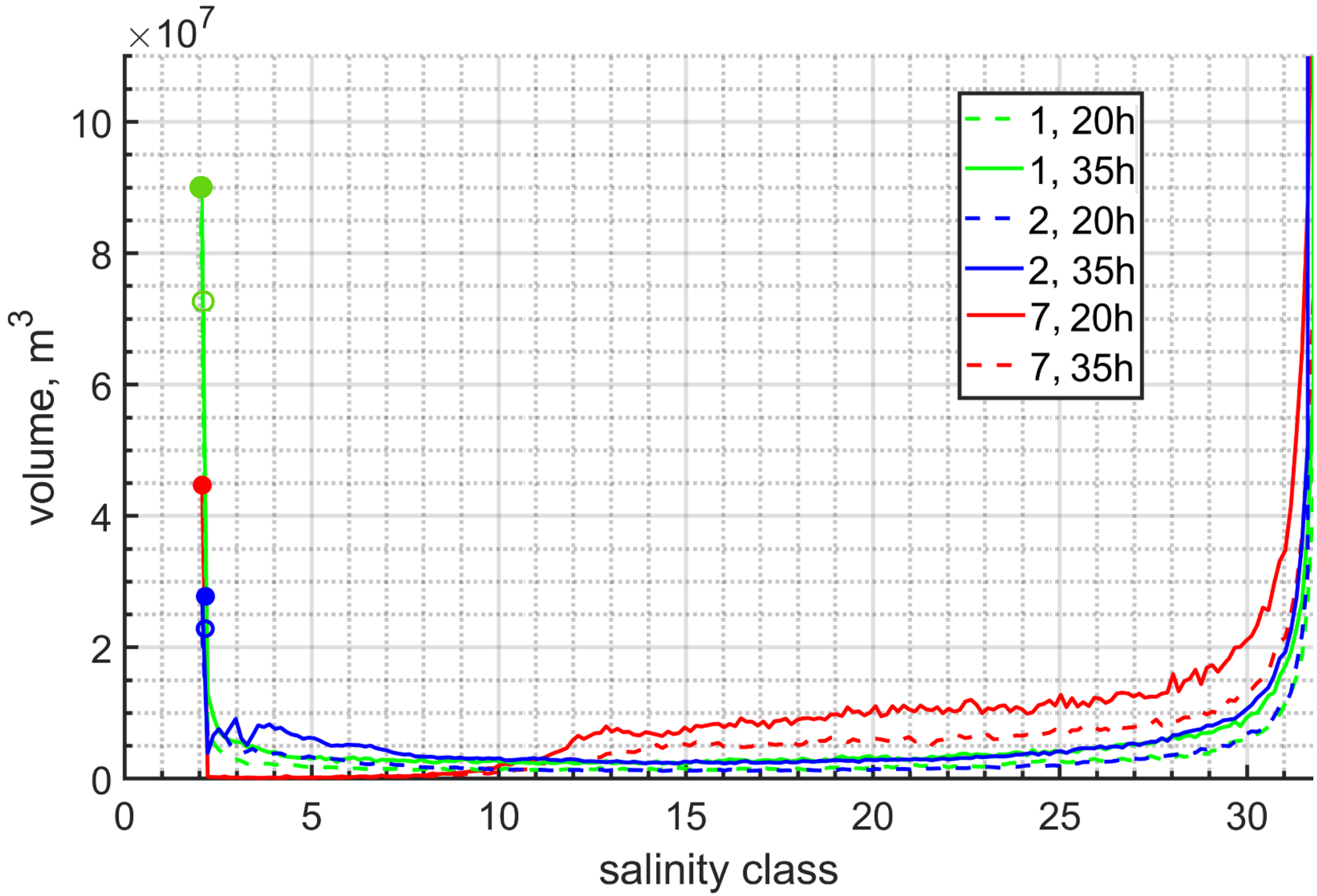
Runs

Number	Adv. Scheme	Turbulence closure for tracer equation	Limiter	Turbulence closure for momentum	N of vertical sigma levels with parabolic distr.	Model/grid
1 (default)	85% of 3rd order + 15% of 4th order (MUSCL type)	off	fct1	$k-\epsilon$	41	FESOM-C/tri
2	2nd order (upwind)	off	geom.	$k-\epsilon$	41	Thetis/tri
3	MUSCL	off	fct2	$k-\epsilon$	41	FESOM-C/tri
4	MUSCL	off	fct3	$k-\epsilon$	41	FESOM-C/tri
5	2nd order (Miura)	off	fct1	$k-\epsilon$	41	FESOM-C/tri
6	2nd order (upwind)	off	no	$k-\epsilon$	41	FESOM-C/tri
7	MUSCL	on	fct1	$k-\epsilon$	41	FESOM-C/tri
8	MUSCL	off	fct1	const. vertical eddy visc. coeff., m^2/s: a) 3e-4 b) 1e-3 c) 0.1	41	FESOM-C/tri
9	MUSCL	off	fct1	$k-\epsilon$	21	FESOM-C/tri
10	2nd order (Miura)	off	fct1	$k-\epsilon$	21	FESOM-C/tri
11	MUSCL	off	fct1	$k-\epsilon$	41	FESOM-C/quad
12	2nd-order (TVD)	off	superbee	$k-\epsilon$	41	GETM/quad
13	3d order HSIMT (TVD)	off	Sweby's	$k-\epsilon$	41	GETM/quad

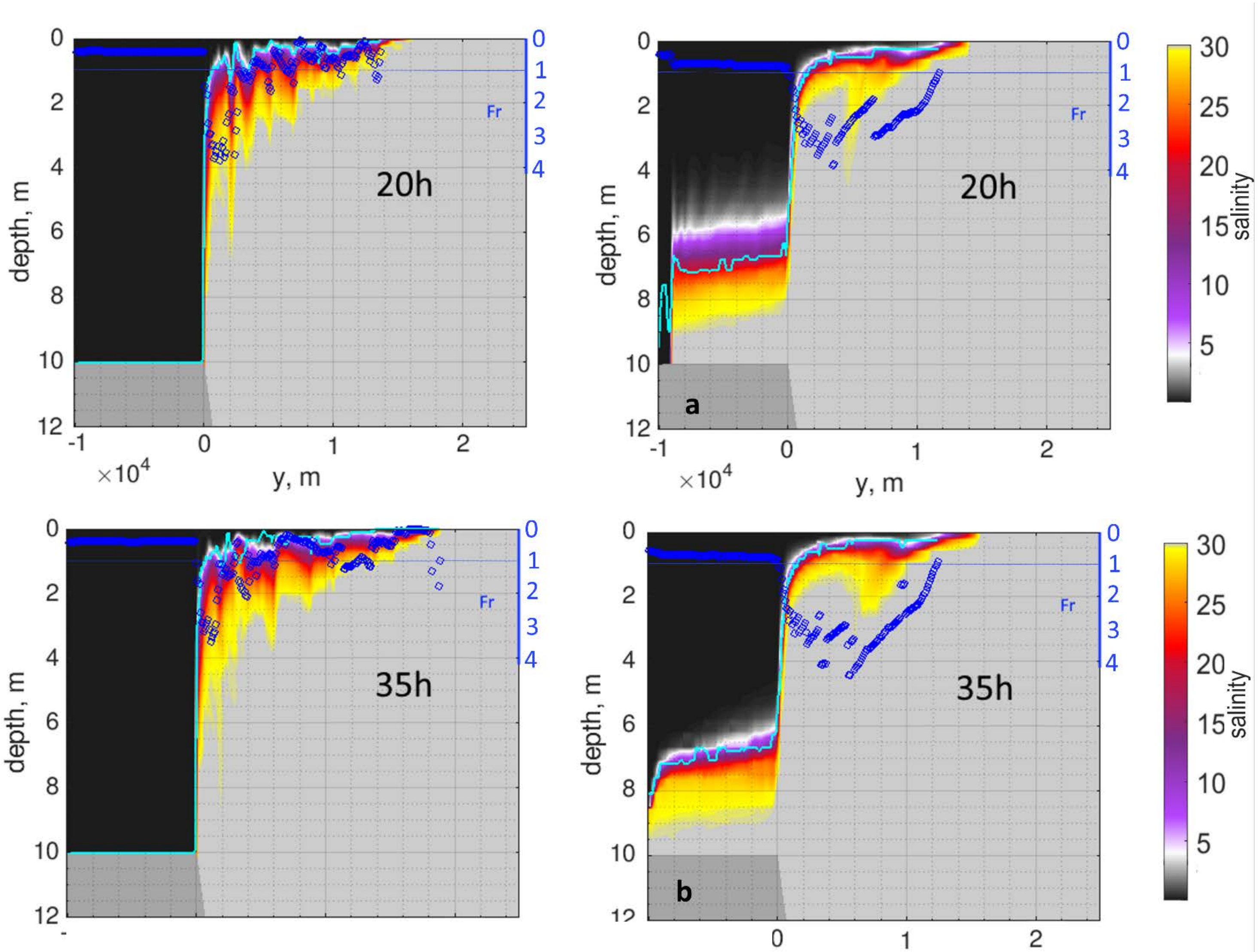


Performance on rectangular grid☺)

Salinity classes-volume diagram



Froude numbers



Setup details: avoid initial schok

How to predict the behaviour?

Flow in a steady cyclostrophic balance

$$-\frac{{v_c}^2}{r} - f v_c = -g' * d_r,$$

v_c - azimuthal cyclostrophic velocity (at radius r), d - thickness of the buoyant layer

g' - reduced gravity, f - Coriolis parameter, r is the radial distance from the anticyclone

Thickness is nearly uniform from the source till the center of the anticyclone and gradually decreases to zero along the outer edge.

The radial change in plume thickness:

$$d_r = -h_b/r \quad \Rightarrow$$

$$r = -(g'h_b + {v_c}^2)/(f v_c)$$

Bernoulli function for the buoyant layer (Gill, 1982):

$$B = g'd + \frac{v^2}{2}$$

It should be constant along the streamline.

$$g'h_? + {v_?}^2/2 = \frac{{v_c}^2}{2}$$

$$v_c = -\sqrt{2g'h_? + {v_?}^2}$$

$$2r = \frac{2(g'h_b + 2g'h_{in} + {v_{in}}^2)}{f\sqrt{2g'h_{in} + {v_{in}}^2}} \approx 24 \text{ km}$$