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Wage bargaining and employment revisited: separability and efficiency in collective bargaining*

Claus-Jochen Haake
Paderborn University, DE-33098 Paderborn, Germany
cjhaake@wiwi.upb.de

Thorsten Upmann†
HIFMB at the University of Oldenburg, DE-23129 Oldenburg, Germany
tupmann@hifmb.de

Papatya Duman
Paderborn University, DE-33098 Paderborn, Germany
pduman@wiwi.upb.de

Abstract

We analyse the two-dimensional Nash bargaining solution (NBS) by deploying the standard labour market negotiations model of McDonald and Solow. We show that the two-dimensional bargaining problem can be decomposed into two one-dimensional problems, such that the two solutions together replicate the solution of the two-dimensional problem if the NBS is applied. The axiom of "independence of irrelevant alternatives" is shown to be crucial for this type of decomposability. This result has significant implications for actual negotiations because it allows for the decomposition of a multi-dimensional bargaining problem into one-dimensional problems – and thus helps to facilitate real-world negotiations.

Keywords: Efficient bargains; labour market negotiations; Nash bargaining solution; restricted bargaining games; sequential bargaining

JEL classification: *C*78; *J*41; *J*52

1. Introduction

In 2018, over 82 million workers were members of trade unions in OECD countries and "about 160 million workers were covered by collective agreements concluded either at the national, sectoral, occupational or firm level" (see OECD, 2019a). Despite the fact that the share of workers organized in trade unions has been declining, these figures show that the study of

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†Also affiliated with Carl von Ossietzky University Oldenburg and Bielefeld University.

collective bargaining as a labour market institution has not lost any of its importance, and research on an efficient functioning of these institutions is still a relevant issue.

The particular labour market institutions that provide the framework to collective labour market negotiations are quite diversely organized across OECD countries, and are thus highly country-specific. Several empirical works, mainly focusing on OECD countries, have surveyed existing labour market institutions, providing a taxonomy of these institutions, elaborating on different systems of negotiations, and analysing the effects of collective bargaining on labour market performance (see, e.g., Du Caju et al., 2008; International Labour Office, 2015; OECD, 2019b, 2019a; Garnero, 2021). The degree of (de-)centralization of labour market negotiations predominant in each country is a distinctive feature. Systems range from fully decentralized (e.g., in Poland, the UK, and the US), where bargaining mainly takes place on the firm level; to organized decentralized, with a combination of negotiations on a sectoral and on a firm level (e.g., Austria, Germany, and the Scandinavian countries); and to centralized (e.g., Belgium, Iceland, and Italy) (see OECD, 2019a, Table 2.10). Specifically, the combination of sectoral- and firm-level negotiations typically separates the issues under negotiation in organized decentralized systems. On the sectoral level, wages are the dominant issue, while employment issues (e.g., workforce, manning rules, job guarantees, hours of overtime, etc.) are left to firm-level negotiations. This, by no means, implies that negotiations on wages and employment are carried out independently – they are clearly interdependent.

In accordance with the broad range of existing labour market institutions, the literature has established a variety of labour market models that use both cooperative and non-cooperative approaches. Starting with the ground-breaking work of McDonald and Solow (1981), solution concepts from axiomatic bargaining theory have been used to model the outcome of the bargaining problem on the labour market. Specifically, in their efficient bargains model, McDonald and Solow (1981) applied the bargaining solution of Nash (1950) – NBS hereafter – to propose an efficient agreement for the two-dimensional bargaining problem on the wage rate and the employment level. Consequently, McDonald and Solow (1981) popularized the NBS among labour economists, such that, in the aftermath, the NBS was almost exclusively used as “the” solution concept (of cooperative bargaining theory) to model the outcome of labour market negotiations.

Even after about 40 years of work on the NBS in labour markets, the analysis is (as we shall show here) still incomplete. In this paper, therefore we complement this work and contribute to the analysis of the NBS in two ways: first, we present a rigorous analysis of the efficient bargains model and its bargaining theoretic foundation; and second, we work out and exploit the structural properties of the NBS, which enables us to propose new bargaining agendas that lead to an efficient outcome. We start in Section 3 by recapitulating the original two-dimensional bargaining problem as already formulated in McDonald and Solow (1981), which we complement with two (to our knowledge) new lemmas (Lemmas 1 and 2) and by an explicit proof of a structural property of the Pareto curve that was merely mentioned by McDonald and Solow (1981) (Lemma 3). These results shed more light on the structure of workers' utilities and, consequently, on the location of efficient contracts (i.e., of the Pareto curve). The rigorous and coherent formal treatment with a novel, fully fledged diagrammatic illustration addresses a lack that has occasionally led to flawed figures in the literature – see, for example, Svejnar (1986, figure 2) and Paz Espinosa and Rhee (1989, figure 1).

In our analysis of the model, we carefully scrutinize the structural properties of the NBS on the labour market. In Section 4, this allows us to identify the two-dimensional bargaining problem on both the wage rate and the employment level as a composition of the two corresponding one-dimensional problems on either the wage rate or the employment level. The merits of a thorough application of the game-theoretic solution concepts to the economic problem become clear in our diagrammatic analysis, where we display the bargaining problem(s) in both the space of physical outcomes (i.e., the wage rate and the employment level) and in the utility space. By decomposing the two-dimensional bargaining problem into two one-dimensional bargaining problems, we reveal the relationship between the NBSs of the former and the latter. To this end, we apply the NBS to each of these one-dimensional bargaining problems, where the parties either negotiate the wage rate (w) or the employment level (L), while taking the other variable as given. In the first case, we obtain a family of solutions that are parametrized in L; in the second case, we obtain a family parametrized in w. We refer to these two parametrized solutions as the w-Nash and the L-Nash curves, respectively. While the latter basically appears in the literature as a technical device – namely as a "solution" of the first-order condition for the employment level, used to find the NBS of the efficient bargaining problem (see, e.g., McDonald and Solow, 1981; Creedy and McDonald, 1991; Bayındır-Upmann and Raith, 2003) – the former has not yet been recognized in the literature.[1] However, as we show here, both of these curves represent collections of solutions to the corresponding one-dimensional bargaining problems.

Our approach to decompose a multi-dimensional problem and to keep track of the bargaining solutions of the corresponding lower-dimensional problems

[1]The literature uses a varying nomenclature to denote, what we call here, the L-Nash curve: McDonald and Solow (1981) refer to this curve as the equity locus, Creedy and McDonald (1991) and Goerke (1996) call it the power locus, while Bayındır-Upmann and Raith (2003, 2005) refer to it as the Nash curve.

is not only limited to the NBS but can also be pursued for any bargaining solution concept – eventually leading to a w-solution and an L-solution curve for the labour market problem. While this is true for any solution concept, in Section 5 we demonstrate that for the NBS the intersection of the w-solution curve and the L-solution curve coincides with the solution of the efficient bargains model, where the parties simultaneously negotiate the wage rate and the employment level (Propositions 1 and 2). The reason underlying this structural property is that the NBS satisfies the axiom of independence of irrelevant alternatives (the IIA axiom). Although this axiom is constitutive for the NBS, its salient significance has apparently been left unnoticed in the literature on wage bargaining. When adapted to our set-up, a solution of the multi-dimensional bargaining problem that satisfies the IIA axiom remains unaltered when some quantities are fixed to the level in the solution, and bargaining only takes place over the remaining quantities. So, one can also view the IIA axiom as a consistency property that imposes stability on the agreement in the sense that further negotiations on any quantity will lead to the same agreement, and hence to efficiency. Conversely, bargaining solutions that do not satisfy this axiom, such as the Kalai–Smorodinsky solution, can generically not be decomposed in the way that was described above, and they therefore fail to reach an efficient outcome via sequential negotiations.

Besides these theoretical results, in Section 6 we address the more practical question of how our insights can help to find efficient agreements for the two-dimensional problem. Using our results on the decomposability of the NBS, we propose that sequential (Proposition 3) and iterative (Proposition 4) one-dimensional negotiation agendas can be designed to arrive at the bargaining outcome of the (static) two-dimensional bargaining problem, and thus can bring about the globally efficient bargaining outcome. From a policy point of view, the implications of our (theoretical) analysis can be summarized as follows:

(i) splitting the two-dimensional wage bargaining problem into a sequence of one-dimensional bargaining problems helps to reach efficient outcomes;

(ii) special care has to be taken when deciding which solution should be used to resolve the conflict.

2. Related literature

In addition to the efficient bargains model with agreements on wages and employment, labour market negotiations have alternatively been modelled as negotiations on wages exclusively; while employment is left at the discretion of firms and is set in the sequel to an agreement on wages. When this approach is combined with the NBS, we obtain the "right-to-manage" model, where only the wage rate is determined by the NBS. In contrast to the efficient bargains model, the final outcome in the right-to-manage model is not located on the Pareto curve but is located on the labour demand curve, and is thus generically inefficient.

From an empirical perspective, the question arises of which model is more descriptive for actual labour market institutions. In this regard, the empirical evidence for whether labour market equilibria lie on the labour demand curve or on the Pareto curve (or offside of both) is mixed.[2] This finding is hardly a surprise when given the huge variety of industrial institutional differences. Because there is some empirical evidence in favour of equilibria on the Pareto curve, efficient bargain models appear to be descriptive in these cases. Correspondingly, there is a substantial body of theoretical literature on labour market negotiations that applies the efficient negotiations model: Horn and Svensson (1986), Svejnar (1986), Paz Espinosa and Rhee (1989), Dowrick (1989, 1990), Clark (1990), Bughin (1996), Bayındır-Upmann and Raith (2003, 2005), Kraft (2006), Upmann (2009), Dittrich (2010), Walsh (2012), Eichner and Upmann (2012, 2014), Fanti and Gori (2013), Upmann and Müller (2014), Müller and Upmann (2018), and others document this significance of the efficient bargains model. Nevertheless, the larger part of the theoretical literature on labour market negotiations presupposes that labour market equilibria lie on the labour demand curve, and thus adopt the simple one-dimensional right-to-manage model.[3]

In view of this overwhelming presence of the NBS in the literature and its proven empirical relevance, it is indispensable to thoroughly study the properties of the NBS within the particular situation of labour market negotiations. In this regard, the articles of McDonald and Solow (1981), Oswald (1985), Binmore et al. (1986), and Creedy and McDonald (1991) are fundamental; however, Alexander and Ledermann (1994, 1996) also conceptually contribute in two frequently overlooked articles that examine the properties of the NBS in labour markets. Since then, the focus of conceptual interest in the NBS has shifted towards the issue of

[2]For a review of the empirical literature, the interested reader is referred to the overviews of Aidt and Tzannatos (2008), Lawson (2011), and Cahuc et al. (2014, Chapter 7.4), who provide valuable surveys of the literature. In particular, a list of articles that find empirical evidence in favour of efficient bargains can be found in Upmann and Müller (2014, p. 339).

[3]See, for example, Zhao (1995), Dutt and Sen (1997), Petrakis and Vlassis (2000), Grandner (2001), Strand (2002), Albrecht and Vromen (2002), Flinn (2006), Boeri and Burda (2009), Wehke (2009), Gertler and Trigari (2009), Belan et al. (2010), Krusell et al. (2010), Creane and Davidson (2011), Ranjan (2013), Santoni (2014), Colciago and Rossi (2015), and many others.

agenda setting and pattern bargaining in labour markets: Dowrick (1990), Dobson (1994), Bughin (1999), Petrakis and Vlassis (2000), and Creane and Davidson (2011) are examples for this strand of the literature. These authors investigate the (strategic) choice of the set of variables to negotiate (agenda setting) and the sequence of issues to be negotiated (pattern bargaining).

Finally, there are only few works that do not deploy the NBS to resolve the bargaining problem. The only exceptions that we are aware of are Alexander (1992), Alexander and Ledermann (1996), Gerber and Upmann (2006), Amine et al. (2009), Dittrich (2010), Dittrich and Knabe (2013), l'Haridon et al. (2013), Liu et al. (2017), and Amine et al. (2018), all of whom apply the Kalai–Smorodinsky solution (Kalai and Smorodinsky, 1975) to model the outcome of labour market negotiations; while Jacquet et al. (2014) apply the egalitarian solution. The remaining literature deploys the NBS to model labour market negotiations (or use a non-cooperative approach). Thus, there is a tremendous dominance of the NBS in models of labour market negotiations, which is the reason why we decided to elaborate on this issue.

3. The model

We deploy the well-known labour market bargaining model that was elaborated by McDonald and Solow (1981) to study labour market negotiations between an employer (i.e., a firm) and a trade union representing the interests of all workers. The firm behaves competitively on the output market and produces its product by means of labour (and some fixed factor). Both parties negotiate a labour contract by specifying the wage rate and the employment level. We subsequently characterize both parties' preferences over possible agreements and the resulting Pareto-efficient allocations. While in this section we focus on an extensive discussion of the Pareto curve, in the the next section we formulate the resulting bargaining problem(s) in utility space.

The typical feature of this model is that it takes more than one bargaining dimension into account and thus fits to observations of a broader scope of labour market negotiations. In their review, Hayter et al. (2011) argue that the "collective bargaining agenda has expanded in many parts of the world"; namely, in OECD countries as well as in developing countries.[4] For the theoretical analysis, we direct the reader's attention to the two dimensions (wage and employment) that are used in McDonald and Solow (1981).

[4] A survey of recent collective bargaining agreements in Germany can be found in Schulten (2020).

3.1. Labour supply

The supply side of the labour market consists of a mass N of worker households. For ease of presentation, we subsequently speak of N as the number of worker households or workers. Suppose that workers have identical preferences and productivity, so that they are homogeneous regarding all aspects of interest. An employed worker derives utility $v : \mathbb{R}_+ \to \mathbb{R}_+$ from the received wage w. We assume $v(0) = 0, v' > 0, v'' \leq 0$. An unemployed worker obtains a fixed utility level $\overline{v} \geq 0$, which represents the utility from unemployment benefits and leisure time. Suppose that there exists some wage rate $\overline{w} \geq 0$ with $v(\overline{w}) = \overline{v}$, so that $\overline{w} := v^{-1}(\overline{v})$ is the reservation wage, below which a worker is not willing to work.[5] Consequently, aggregate labour supply equals zero for all $w < \overline{w}$, equals N for all $w > \overline{w}$, and is indeterminate for $w = \overline{w}$; that is,

$$L^s(w) = \begin{cases} N & \text{if} \quad w > \overline{w}, \\ [0, N] & \text{if} \quad w = \overline{w}, \\ 0 & \text{if} \quad w < \overline{w}. \end{cases} \tag{1}$$

Following Oswald (1985), the trade union represents the interests of all workers, so that its utility depends on the negotiated wage w and employment level L (considered as a continuous variable) and is given by $u_1 : \mathbb{R}_+ \times [0, N] \to \mathbb{R}$,

$$u_1(w, L) := Lv(w) + (N - L)v(\overline{w}).$$

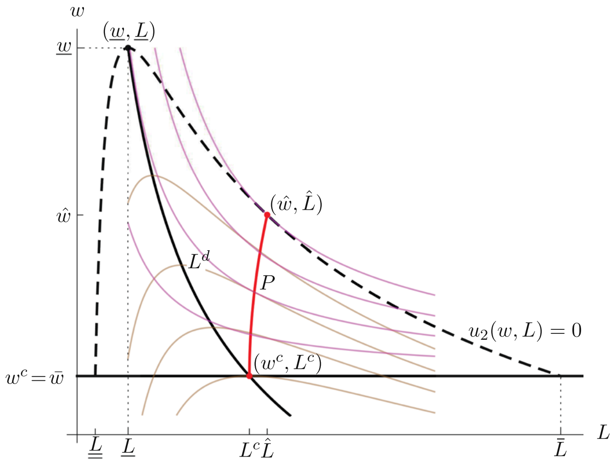
The utility of the labour union can thus be viewed as the aggregate utility of its members. Differentiation of u_1 with respect to w and L gives the marginal rate of substitution between wage and employment,

$$\left.\frac{dw}{dL}\right|_{u_1 \text{ const.}} = -\frac{\partial u_1(w, L)/\partial L}{\partial u_1(w, L)/\partial w} = -\frac{v(w) - \overline{v}}{Lv'(w)} = -\frac{w}{L}\frac{1}{\sigma(w)}, \tag{2}$$

where $\sigma(w) := (wv'(w))/(v(w) - \overline{v})$ denotes the elasticity of the excess utility (the utility above the reservation utility $\overline{v}$). Because $\sigma(w) > 0$ for all $w > \overline{w}$,

[5] As recently pointed out by Jäger et al. (2020), the non-employment value – mainly ruled by unemployment benefits and the utility of leisure time – determines the reservation wage and thus constitutes a lower bound for an equilibrium wage. However, there are also other outside options, beyond unemployment, that are available to an employed worker, such as alternative employment opportunities in comparable firms (possibly at a lower wage rate), employment at the minimum wage elsewhere in the economy, self-employment and compensation by an increase of the spouse's working time, among other things. Here, we bear in mind all of these sources that might possibly affect the reservation wage.

Figure 1. The union's indifference curves and the firm's iso-profit lines, along with the Pareto and labour demand curves



Notes: The union's indifference curves (magenta), the reservation wage (horizontal line), the firm's iso-profit lines (brown), the zero profit line (dashed curve), the labour demand curve L^d (black curve) and the Pareto curve P (red).

the union's indifference curves are downward-sloping in the area above the reservation wage $\overline{w}$, and thus for all relevant wage rates, as illustrated in Figure 1. (To have figures that are in line with those in the literature, we display the wage on the vertical axis and the employment level on the horizontal axis throughout this paper, but maintain the order of the variables as defined in the utility functions.) The trade union is indifferent between all agreements (w, L) with either $w = \overline{w}$ or $L = 0$, as all of those yield $u_1(\overline{w}, L) = u_1(w, 0) = N\overline{v}$, for all w, L.

By defining the elasticity of the marginal utility by $\varepsilon(w) := [wv''(w)]/v'(w) \leq 0$, we find a surprisingly simple characterization of the slope of the elasticity of the excess utility.

Lemma 1. *For any $w > \overline{w}$, the elasticity of the excess utility σ satisfies*

$$\sigma'(w) \gtreqless 0 \quad \Leftrightarrow \quad \sigma(w) \lesseqgtr 1 + \varepsilon(w).$$

As well as all other proofs, the proof of Lemma 1 is relegated to Appendix B.

While Lemma 1 applies to any $w > \overline{w}$, we show below that $\sigma(w) > 1$ for any w in the neighbourhood of the Pareto curve (Lemma 2).

3.2. Labour demand

The firm produces its output by means of a technology featuring decreasing returns to scale, with production function $f : \mathbb{R}_+ \to \mathbb{R}_+$ satisfying $f(0) = 0$,

$f' > 0$, $f'' < 0$. Normalizing the output price to unity, the profit obtained by employing L workers at wage rate w is given by $u_2 : \mathbb{R}_+ \times [0, N] \to \mathbb{R}$ such that

$$u_2(w, L) := f(L) - wL - \mathbb{1}_{\{L:L>0\}}C,$$

where $\mathbb{1}_A$ denotes the indicator function for event A, and $C \geq 0$ denotes the set-up cost incurred upon start of production (i.e., if some positive amount of labour is deployed). (The limit case $C = 0$ is analysed in Appendix A.) Thus, we have $u_2(w, 0) = 0$, but $\lim_{L \searrow 0} u_2(w, L) < 0$ if $C > 0$. In other words, when $C > 0$, there is a minimum number of labour $\underline{\underline{L}}$ to ensure non-negative profits, so $u_2(\overline{w}, \underline{\underline{L}}) = 0$ and $u_2(\overline{w}, L) < 0$ for all $L < \underline{\underline{L}}$. Moreover, we assume that labour supply is large enough (or the reservation wage is high enough) to ensure that not all workers can be profitably employed.[6]

Assumption 1. *Not all workers can be employed at non-negative profits:*

$$u_2(\overline{w}, N) = f(N) - N\overline{w} - C < 0,$$

and there exists an employment level $L \in [0, N]$ *with* $u_2(f'(L), L) > 0$.

Assumption 1 implies that there is some employment level $\overline{\overline{L}} < N$ such that $u_2(\overline{w}, \overline{\overline{L}}) = f(\overline{\overline{L}}) - \overline{\overline{L}}\overline{w} - C = 0$ and $u_2(\overline{w}, L) < 0$ for all $L > \overline{\overline{L}}$. Accordingly, any reasonable employment level lies above $\underline{\underline{L}}$ but below $\overline{\overline{L}}$. Moreover, for each employment level L, let $w^0(L)$ represent the wage level that yields zero profit; that is, $w^0(L) = (f(L) - C)/L$. The graph of $w^0(L)$ is represented by the dashed line in Figure 1.

By differentiating u_2 with respect to w and L, we obtain the slope of the iso-profit curves:

$$\left.\frac{dw}{dL}\right|_{u_2 \text{ const.}} = -\frac{\partial u_2(w, L)/\partial L}{\partial u_2(w, L)/\partial w} = -\frac{w - f'(L)}{L}. \tag{3}$$

Each iso-profit curve increases in L until $f'(L) = w$, and it then decreases afterwards (as displayed in Figure 1). Accordingly, for any given wage rate, the profit-maximizing employment level is located where the iso-profit curve is horizontal. As a minimum amount of labour is required to ensure non-negative profits, there is a maximum wage that the firm is willing to pay: the wage rate $\underline{w} > \overline{w}$ defined as the root of $u_2(\cdot, (f')^{-1}(\cdot))$. Hence, when employment

[6]For most of our results, this assumption is not necessary and it would suffice to assume that N exceeds the largest efficient employment level $\hat{L}$ (see the definition given by equation (6)), so that N does not limit the Pareto curve.

is unprofitable, labour demand equals zero, and is characterized by $f'(L) = w$ else, so that labour demand is given by

$$L^d(w) = \begin{cases} (f')^{-1}(w) & \text{if} \quad 0 \leq w \leq \underline{w}, \\ 0 & \text{else.} \end{cases} \tag{4}$$

Given that L^d is decreasing, the minimum number of labour on the labour demand curve is obtained for the maximal wage rate: $\underline{L} := L^d(\underline{w})$. So, the "origin" (or the "upper end") of the labour demand curve is located at $(\underline{w}, \underline{L})$ (see Figure 1). For the limiting cases $f'(0) \to \infty$ and $C = 0$, we have $\underline{w} \to \infty$ (see Appendix A). Henceforth, we restrict our attention to $\mathcal{L} := [\underline{L}, \overline{L}]$ and $\mathcal{W} := [\overline{w}, \underline{w}]$ because these intervals cover all reasonable employment levels and wage levels.

3.3. Pareto-efficient allocations

We can identify Pareto-efficient labour contracts using the characterizations of the parties' preferences; that is, efficient pairs of a wage rate and an employment level (w, L) (as depicted in Figure 1). These contracts can be obtained by equating the parties' marginal rates of substitution, given by equations (2) and (3), yielding

$$\sigma(w) = \frac{w}{w - f'(L)}. \tag{5}$$

Equation (5) equates the elasticity of excess utility with the reciprocal of the wage markup above the marginal productivity of labour given by $(w - f'(L))/w$. Using this characterization of Pareto-efficient contracts, we find the following.

Lemma 2. *In the neighbourhood of the Pareto curve, the elasticity of (excess) utility exceeds unity; that is,* $\sigma(w) > 1$.

It follows from the combination of Lemma 1 and 2 that the elasticity of the excess utility σ decreases in the neighbourhood of the Pareto curve.

The competitive labour market equilibrium is obtained by equating labour supply (equation (1)) and labour demand (equation (4)), which yields the pair $(w^c, L^c) = (\overline{w}, L^d(\overline{w}))$. Clearly, the competitive equilibrium is Pareto-efficient because it satisfies equation (5). While the competitive equilibrium leaves the labour union with its reservation (or status quo) utility level, the employer obtains the maximum feasible profit:

$$u_1^c := u_1(w^c, L^c) = N\overline{v}, \qquad u_2^c := u_2(w^c, L^c) = f(L^c) - \overline{w}L^c - C > 0.$$

In other words, u_2^c is the maximal feasible profit when the utility of the labour union is at least $N\overline{v}$. Formally,

$$u_2^* := \max_{w \geq 0,\ 0 \leq L \leq N} u_2(w, L) \quad \text{s.t.} \quad u_1(w, L) \geq N\overline{v}.$$

It is straightforward to see that the constraint is binding at the solution, and hence the unique feasible utility allocation most favourable for the firm is induced by the pair $(w^c, L^c) = (\overline{w}, L^d(\overline{w}))$, yielding $u_2^* = u_2^c$.

Similarly, the individually rational utility level most favourable to the labour union, u_1^*, is achieved by the pair $(\hat{w}, \hat{L})$, which solves

$$u_1^* := \max_{w \geq 0,\ 0 \leq L \leq N} u_1(w, L) \quad \text{s.t.} \quad u_2(w, L) \geq 0. \tag{6}$$

It is immediate to verify that firms are left with zero profits; that is, $u_2(\hat{w}, \hat{L}) = f(\hat{L}) - \hat{w}\hat{L} - C = 0$. Moreover, $(\hat{w}, \hat{L})$ must be Pareto-efficient; that is, it must satisfy equation (5).[7] Hence, $u_1^* = u_1(\hat{w}, \hat{L})$. Consequently, w^c and $\hat{w}$ are the lowest and the highest Pareto-efficient wage rates, respectively; and L^c and $\hat{L}$ are the lowest and the highest Pareto-efficient employment levels, respectively. Therefore, we define the sets $\mathcal{L}^\circ := [L^c, \hat{L}]$ and $\mathcal{W}^\circ := [w^c, \hat{w}]$, representing the range of all individually rational, Pareto-efficient employment and wage levels, respectively. We then define the function $\Phi^P : \mathcal{W}^\circ \times \mathcal{L}^\circ \rightarrow \mathbb{R}$ by

$$\Phi^P(w, L) := \sigma(w) - \frac{w}{w - f'(L)}.$$

Thus, the roots of Φ^P are the Pareto-efficient pairs (w, L). Accordingly, we implicitly define the Pareto curve $P : \mathcal{L}^\circ \rightarrow \mathcal{W}^\circ$ by $\Phi^P(P(\cdot), \cdot) = 0$; that is, $w = P(L)$ is root of $\Phi^P(., L)$.

Lemma 3. *The slope of the Pareto curve P is positive with $P'(L^c) = +\infty$; that is, the Pareto curve is vertical at its "lower end" $(w^c, L^c) = (\overline{w}, L^d(\overline{w}))$.*

While the first part of Lemma 3 is well established in the literature (e.g., Dowrick, 1989, p. 1129), the second part is mentioned in McDonald and Solow (1981), yet without a formal proof. It says, first, that the Pareto curve is vertical at the competitive equilibrium in a w–L diagram and, second, that this result holds for any type of the workers' utility function (as long as $\varepsilon(w^c)$ is finite); that is, irrespective of whether utility function is concave or convex. We point to this feature of the Pareto curve because it has sometimes been left

[7] To see this, observe that the constraints $0 \leq L \leq N$ are not binding at $\hat{L}$, because $u_1(w, L^c) > N\overline{v}$ for $w = (f(L^c) - C)/L^c$ and $u_1(w, N) < N\overline{v}$ for $w = (f(N) - C)/N$ as $(f(N) - C)/N < \overline{w}$ by Assumption 1.

unnoticed in the literature and has thus led to flawed figures; see, for example, Svejnar (1986, figure 2) and Paz Espinosa and Rhee (1989, figure 1).

4. The bargaining model

While in the previous section we characterized the possible combinations of a wage rate and an employment level that can emerge as an outcome of the bargaining problem, we now discuss the utilities (or payoffs) induced by these outcomes. More precisely, we extend our view of the problem by considering the utility allocations resulting from feasible (w, L) agreements. Hence, we formulate an axiomatic bargaining problem, which goes back to the ground-breaking work of Nash (1950).

Apart from discussing the bargaining problem originating from negotiations over both quantities, we also consider those bargaining problems in which one of the quantities (wage or employment level) is treated as fixed, and negotiations take place only over the other. Examples for such one-dimensional bargaining problems can be found in the public sector. For instance, in Germany, wages for civil servants are regulated by law and are therefore basically excluded from negotiations (see International Labour Office, 2015). This can be taken as a descriptive motivation to draw special attention to one-dimensional bargaining problems, which we call restricted bargaining problems. Section 5 is devoted to the study of the Nash bargaining solutions of these restricted problems.

4.1. The abstract bargaining problem

A two-person bargaining problem is a pair (S, d), where S is a closed, convex, and comprehensive[8] subset of $\mathbb{R}^2$, $S_d := S \cap (d + \mathbb{R}^2_+)$ is bounded and $d \in S$. The set S is called the bargaining set and can be interpreted as a collection of all of the utility allocations resulting from feasible agreements among the two parties, to which we refer here as players, in accordance with the game-theoretic terminology. The point $d \in S$, which is termed the disagreement point (or status quo point), reflects the utility allocation that either becomes effective when negotiations breakdown or is realized during negotiations, depending on the specific interpretation of the model.[9] Because neither player can be forced to sign an unfavourable contract (i.e., a contract

[8] A set $T \in \mathbb{R}^2_+$ is comprehensive, if for all $x \in T$, $y \leq x$ implies $y \in T$.

[9] As thoroughly explained by Binmore et al. (1986), the interpretation of d as an alternative labour market contract (e.g., the wage obtained in an alternative firm/industry, unemployment benefits, etc.) is most appropriate within a risk-of-breakdown model where the opportunity for beneficial negotiations can stochastically disappear due to random, unforeseen, exogenous

that brings about a result worse than the disagreement point), only individually rational points $x \in S$ (i.e., $x \geq d$) are reasonable agreements because neither player strictly prefers to let negotiations fail. Then, a bargaining solution on a set $\mathcal{B}$ of two-person bargaining problems is a mapping $F : \mathcal{B} \to \mathbb{R}^2$ with $F(S, d) \in S$ for all $(S, d) \in \mathcal{B}$. In general, a bargaining solution constitutes, in some reasonable sense, a "fair" way to share common gains (above d). Although there is a large variety of bargaining solutions in the literature, the solution proposed by Nash (1950) is arguably the most prominent; in particular, it is almost exclusively used in the labour market literature (as documented in the Introduction).

In the Nash bargaining solution, the product of excess utilities (i.e., the utilities above d) is maximized. It is immediate that the solution point necessarily has to be Pareto optimal and individually rational. From a descriptive point of view, maximization of the product naturally involves balancing the factors, so that the final utility allocation is located "in the middle" of the Pareto boundary of S. Formally, the Nash bargaining solution $F^N(S, d)$ of a bargaining problem (S, d) is the unique point in S maximizing the product of excess utilities above the disagreement point:[10]

$$F^N(S, d) := \text{argmax}_{(x_1, x_2) \in S_d} \ (x_1 - d_1)(x_2 - d_2). \tag{7}$$

Besides these descriptive arguments supporting the Nash solution, Nash (1950) provides an axiomatic foundation for his solution concept. Apart from the axioms of Pareto optimality, individual rationality, symmetry and scale covariance, the crucial axiom (which will play a key role in our later analysis) is the IIA – a bargaining solution F satisfies the axiom of the independence of irrelevant alternatives, if for any two bargaining problems (S, d) and (T, d) with $S \subseteq T$ and $F(T, d) \in S$ we have $F(S, d) = F(T, d)$.

Thus, suppose that there are two distinct bargaining problems with the same disagreement point, (S, d) and (T, d), but with "more" allocation possibilities in T (i.e., $S \subseteq T$). Then, if the solution $F(T, d)$ of the "larger" problem is feasible in the "smaller" problem S, then this point should also be the solution for S. In this sense, the allocations in $T \setminus S$ are irrelevant for the solution in S.

In the welfaristic context, one typically abstracts from the fact that there is a set of "physical" outcomes generating the utility possibility set. Therefore, arguments for or against a particular solution only take the resulting

effects; while the interpretation of d as the utility obtained during a dispute (e.g., the income during the period of strike or lock-off, etc.) is most appropriate within a time-preference model where impatient parties discount future benefits. Both of these interpretations are compatible with our model.

[10]Because the product $(x_1 - d_1)(x_2 - d_2)$ is strictly quasi-concave and S is assumed to be convex, there is exactly one maximizer in equation (7), so that $F^N(S, d)$ is well defined.

utilities into account. In an economic framework, as in the case of labour market negotiations, we are interested in the underlying variables yielding a particular utility allocation. In labour markets, the underlying variables (i.e., wages, employment, unemployment, working hours, etc.) are arguably of more political and economic concern than are the utilities of the trade union and the employers' federation.

Returning to the labour market model of Section 3, we generate different specific two-person bargaining problems that are induced by negotiations over specific economic variables in the following subsection. The solutions of these bargaining problems in the utility space correspond to physical outcomes (w, L) of a wage rate and an employment level. Because any (w, L) pair induces a utility allocation $u(w, L)$, we are able to reformulate the maximization problem (7) as a maximization problem over (sets of) (w, L) combinations.

4.2. One-dimensional bargaining problems of the labour market

We now formulate these two types of restricted bargaining problems. For fixed $L \in \mathcal{L}$, define the two-person bargaining problem $(S^L(L), d^L(L))$ by

$$S^L(L) := \{x \in \mathbb{R}^2 \mid \exists w \in \mathcal{W} : x \leq (u_1(w, L), u_2(w, L))\},$$
$$d^L(L) := d = (N\bar{v}, 0),$$

and for fixed $w \in \mathcal{W}$, define the bargaining problem $(S^w(w), d^w(w))$ by

$$S^w(w) := \{x \in \mathbb{R}^2 \mid \exists L \in \mathcal{L} : x \leq (u_1(w, L), u_2(w, L))\},$$
$$d^w(w) := d = (N\bar{v}, 0).$$

The strict Pareto boundary of S^L has to be included in the set

$$\{x \in \mathbb{R}^2 \mid x = (u_1(w, L), u_2(w, L)), w \in \mathcal{W}\}.$$

The analogous observation is true for S^w. In each (restricted) bargaining problem, we assume that there is no (overall) agreement in the case of a breakdown of negotiations (i.e., no worker is hired, the wage level does not have to be determined and the setup costs C are not effective). Each worker is paid the reservation wage and nothing is produced, which implies that the status quo point is $(N\bar{v}, 0) = d$. In this context, when we consider the "fixed wage level" or "fixed employment level", the fixed variable should be viewed as "tentatively fixed", and in the case of a breakdown of negotiations the "fixation" becomes immaterial. As the disagreement point is independent of the fixed quantity, for simplicity we have $d = d^L(L) = d^w(w)$.

Remark 1. *The disagreement point d is the only individually rational point in the set $S^L(\overline{L})$. All individually rational points in $S^w(\overline{w})$ provide a utility of $N\overline{v}$ for the trade union, and hence no positive excess utility.*

Lemma 4. *(i) For each $L \in \mathrm{Int}(\mathcal{L})$, (S^L, d) is a well-defined two-person bargaining problem. (ii) For each $w \in \mathrm{Int}(\mathcal{W})$, (S^w, d) is a well-defined two-person bargaining problem.*

5. Nash curves

In the following two subsections, we discuss the classes of restricted bargaining problems and their Nash solutions. This analysis will result in defining two Nash curves, depending on which quantity is bargained over. For each of the restricted bargaining problems, we obtain a well-defined Nash curve.

5.1. Fixed wage level and the *L*-Nash curve

Assume that the wage level is fixed to $w \in \mathrm{Int}(\mathcal{W})$, so that parties only negotiate employment levels L in $\mathcal{L}$. For formal reasons, we first exclude the boundaries of $\mathcal{W}$ because, in either of the two cases, one of the parties is not able to realize a strictly positive excess utility. In analogy to Section 3.2, we can rewrite the maximization problem behind the Nash solution as a maximization problem over employment levels in $\mathcal{L}$ and determine the employment level at which the product of excess utilities is maximal. In effect, we determine a physical outcome $(w, L^*(w))$, the utility allocation of which, $u(w, L^*(w)) \in S^w(w)$, is the Nash bargaining solution of the bargaining problem $(S^w(w), d)$. In other words, for fixed $w \in \mathrm{Int}(\mathcal{W})$, employment level $L^*(w)$ is the employment level that is associated with the Nash bargaining solution of the restricted bargaining problem (S^w, d). Formally,

$$L^*(w) \in \mathrm{argmax}_{L \in \mathcal{L}}(u_1(w, L) - d_1)(u_2(w, L) - d_2), \quad w \in \mathrm{Int}(\mathcal{W}). \quad (8)$$

The function $L^*(\cdot)$ is called the L-Nash curve. The L-Nash curve can be continuously extended to the boundary of its domain. While this is not crucial for the formal analysis, we make use of this fact in the exposition of the L-Nash curve. For instance, the following lemma explicitly determines one of its endpoints. By evaluating the first-order condition of equation (8), the Nash curve can be expressed by a function $\Phi^L : \mathcal{W} \times \mathcal{L} \to \mathbb{R}$ with

$$\Phi^L(w, L) = w - \frac{1}{2}\left(f'(L) + \frac{f(L) - C}{L}\right). \quad (9)$$

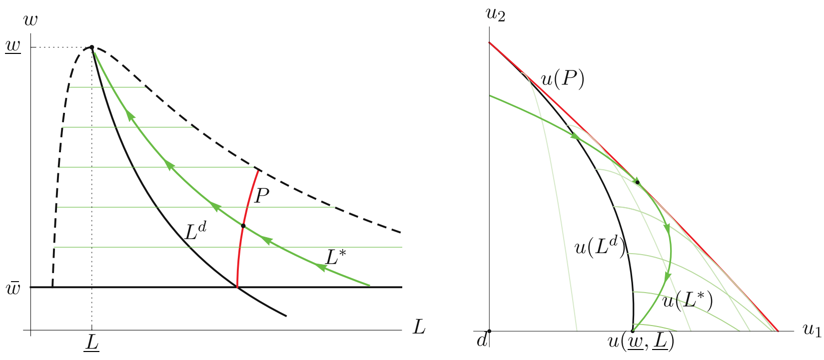
All (w, L) combinations on the L-Nash curve (i.e., $L = L^*(w)$) satisfy $\Phi^L(w, L) = 0$.

Lemma 5. *L^* is downward-sloping for each $w \in \mathcal{W}$ and converges to the point $(\underline{w}, \underline{L})$ as w goes to $\underline{w}$.*

An immediate consequence of $\Phi^L(w, L) = 0$ is that $w = (1/2)((f(L) - C)/L + f'(L))$. Hence, the L-Nash curve can be viewed as the wage rate being a function of L that is constructed as the average of the marginal and average products. This point of view is prevalent in the literature. However, the very construction of the L-Nash curve takes each wage level w to a compromising employment level L, which is given by the NBS of the restricted bargaining problem $(S^w(w), d)$. The L-Nash curve should therefore be seen as given by the function L^*, which depends on the wage level w. The employment level is determined by the wage rate. This means that the wage rate equally weighs the interests of the union (average product of labour) and of the employer (marginal product of labour).

The left panel of Figure 2 displays the L-Nash curve, which is the Pareto curve in the w–L space. For given w, as indicated by a dashed horizontal line, the corresponding employment level on the L-Nash curve marks the Nash bargaining solution of the restricted bargaining problem. Consequently, the right panel of Figure 2 shows the images of the L-Nash curve and the Pareto curve in the utility space. The direction indicated by the arrows corresponds to increasing w. The thin lines depict Pareto frontiers of restricted bargaining problems. The image of the L-Nash curve is thus the collection of Nash bargaining solution points. The image of the Pareto curve is the Pareto frontier of the efficient bargaining problem, and its Nash bargaining

Figure 2. The L-Nash curve in the w–L space (left) and in the utility space (right)



Notes: The left panel shows the L-Nash curve L^* (green), the Pareto curve P (red), the labour demand curve L^d (black), the zero profit line (dashed curve). In the right panel, the images of these curves are shown in the same colours with $u(L^*)$ being the graph of $u(\cdot, L^*(\cdot))$. Each thin green horizontal line in the left panel contains (w, L) combinations with a fixed wage w; its image, depicted in the right panel, shows the Pareto optimal points of the corresponding restricted bargaining problem $(S^w(w), d)$.

solution is the common point of the Pareto frontier and the image of the L-Nash curve.

5.2. Fixed employment level and the w-Nash curve

We now analyse the scenario in which the employment level is fixed to $L \in \text{Int}(\mathcal{L})$ and bargaining only takes place over the wage rate w. By formulating the maximization problem behind the Nash solution of $(S^L(L), d)$ as one over wage levels, we obtain

$$w^*(L) \in \text{argmax}_{w \in \mathcal{W}}(u_1(w, L) - d_1)(u_2(w, L) - d_2). \tag{10}$$

Our interest is focused on this function taking each employment level to the corresponding Nash wage. We call the function w^* the w-Nash curve. The w-Nash curve can be continuously extended to the boundary of its domain, which we mainly use for expositional reasons. Because the w-Nash curve is only given implicitly via the first-order condition of equation (10), we formalize this by defining a function $\Phi^w : \mathcal{W} \times \mathcal{L} \to \mathbb{R}$ such that

$$\Phi^w(w, L) = \sigma(w) - \frac{wL}{u_2(w, L)}. \tag{11}$$

All (w, L) combinations on the w-Nash curve (i.e., with $w = w^*(L)$) satisfy $\Phi^w(w, L) = 0$.

Figure 3 parallels Figure 2 by showing the w-Nash curve and Pareto curve in the w–L space, as well as their images in the utility space. The arrows correspond to increasing L. The image of the w-Nash curve contains the Nash bargaining solution points of all of the restricted bargaining problems (given by the thin lines in the right panel of Figure 3), as well as the one of the efficient bargaining problem.

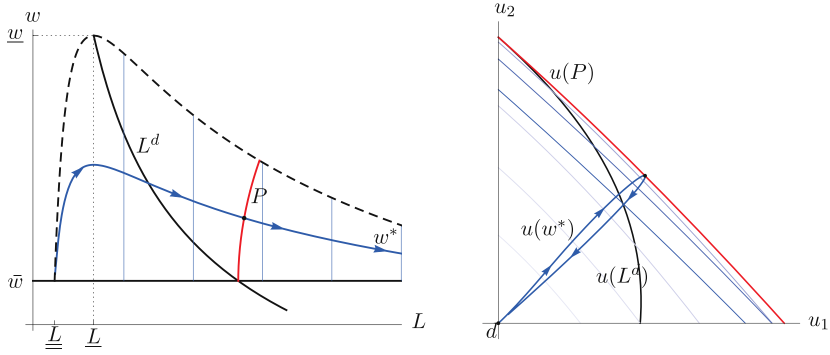
Lemma 6. *w^* is downward-sloping in the neighbourhood of the Pareto curve and converges to the point $(\overline{w}, \overline{L})$ as L goes to $\overline{L}$.*

It follows from Lemmas 2 and 6 that in the neighbourhood of the Pareto curve, the wage share $wL/f(L)$ is greater than the profit share $(f(L) - wL - C)/f(L)$. Because various reasonable utility functions satisfy $\sigma' \leq 0$ (e.g., the utility functions $w^a, \log(w), 1 - (1/w + 1), aw$), we henceforth assume that the elasticity of excess utility is non-increasing.[11]

Assumption 2. *The derivative of the elasticity of excess utility is non-positive for all $w \in \text{Int}(\mathcal{W})$; that is, $\sigma' \leq 0$.*

[11] By imposing this assumption, we follow Creedy and McDonald (1991) and Bayındır-Upmann and Raith (2003).

Figure 3. The w-Nash curve in the w–L space (left) and in the utility space (right)



Notes: The left panel shows the w-Nash curve w^* (blue), the Pareto curve P (red), the labour demand curve L^d (black), the zero profit line (dashed curve). In the right panel the images of these curves are shown in the same colours with $u(w^*)$ being the graph of $u(w^*(\cdot), \cdot)$. Each thin blue vertical line in the left panel contains (w, L) combinations with a fixed employment level L; its image, depicted in the right panel, shows the Pareto optimal points of the corresponding restricted bargaining problem $(S^L(L), d)$.

Corollary 1. *Under Assumption 2, the w-Nash curve is downward-sloping for $L \in \mathcal{L}$; hence, its supremum on $\mathcal{L}$ is attained at $\underline{L}$.*

For any $L \in \mathcal{L}$, let $w^0(L)$ denote the maximum individual rational wage level (i.e., the wage at which the firm's profit is zero). Then, using the definition of Φ^w, for any given $L \in \mathcal{L}$, the negotiated wage w^* can be expressed as

$$w^*(L) = w^0(L)\frac{\sigma(w^*(L))}{1+\sigma(w^*(L))} < w^0(L).$$

The negotiated wage rate can thus be interpreted as a discount of the maximum wage level $w^0(L)$. Consequently, the w-Nash curve is located between the zero profit line and the reservation wage. In particular, it follows from Lemma 2 that in a neighbourhood of the Pareto curve, we have $w^*(L) \geq (1/2)w^0(L)$.

5.3. Location of the Nash curves

In this subsection, we analyse the relative location of the two Nash curves and their intersection. The results are then used in Section 6 to study the outcomes from different bargaining agendas. The next proposition shows that the graphs of the two Nash curves must have a unique intersection point (w^I, L^I). For $L < L^I$, the L-Nash curve is located above the w-Nash curve, while the opposite holds for $L > L^I$.

Proposition 1.

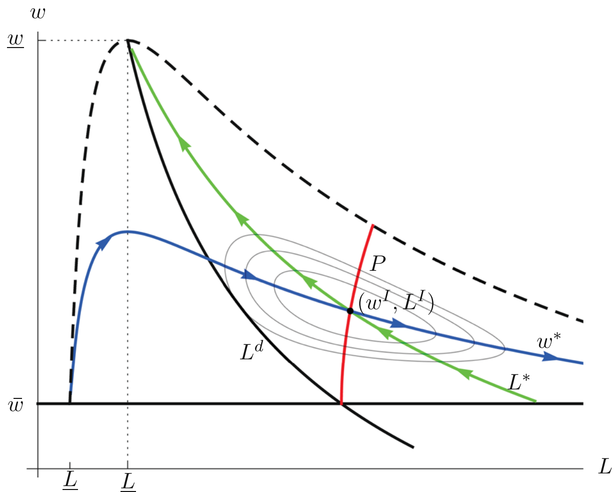
(i) *The w-Nash curve w^* and the L-Nash curve L^* have a unique point of intersection (w^I, L^I).*

(ii) *This intersection point lies on the Pareto curve.*

(iii) *For $L < L^I$, the L-Nash curve is located above the w-Nash curve. For $L > L^I$, the L-Nash curve is located below the w-Nash curve.*

Figure 4 depicts the results in Proposition 1. It shows the two Nash curves in the w–L space and their monotonicity properties, as well as their relative position. Moreover, the unique common point is located on the Pareto curve. The thin curves represent iso-product lines of the Nash product. Due to the technical origins of the Nash curves as solutions of corresponding first-order conditions, the Nash curves intersect the iso-product lines, where the corresponding partial derivative of the Nash product is equal to zero. In other words, the L-Nash curve contains all of the (w, L) combinations at which the iso-product line is horizontal, while the w-Nash curve contains those points at which the iso-product line is vertical.

Figure 4. Results of Proposition 1



Notes: The L-Nash curve (green) and the w-Nash curve (blue) have a common intersection on the Pareto curve (red). The thin ellipsoid-shaped lines depict iso-Nash-product lines, which intersect the L- and the w-Nash curves at the zero and infinite slope, respectively.

5.4. Efficient wage–employment bargaining

Before we take a closer look at the different bargaining protocols, in this subsection we examine the scenario in which the trade union and the firm negotiate the wage and the employment at the same time. In the literature, this scenario is commonly referred to as "efficient bargains". Accordingly, we define the "efficient bargaining problem" (S^e, d) (e.g., McDonald and Solow, 1981) as

$$S^e := \{x \in \mathbb{R}^2 \mid \exists (w, L) \in \mathcal{W} \times \mathcal{L} : x \leq (u_1(w, L), u_2(w, L))\},\ d := (N\bar{v}, 0).$$

For each $L \in \mathcal{L}$, the restricted bargaining problem set $S^L(L)$ is included in S^e. Analogously, for each $w \in \mathcal{W}$, $S^w(w) \subseteq S^e$. We use this fact when applying the IIA axiom to relate the Nash solution of the efficient bargaining problem with the Nash solution of a restricted one. For the efficient bargaining problem (S^e, d), the maximization problem is

$$(w^N, L^N) \in \text{argmax}_{(w,L)\in\mathcal{W}\times\mathcal{L}}(u_1(w, L) - d_1)(u_2(w, L) - d_2). \qquad (12)$$

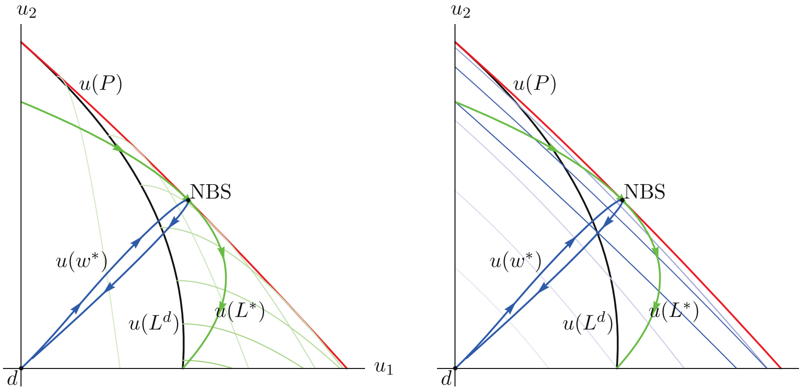
The first-order conditions of equation (12) are precisely those that stem from setting equations (9) and (11) to zero, respectively. This means that they are precisely the first-order conditions that characterize Nash solutions for restricted bargaining problems. As a result, the maximizer (w^N, L^N) coincides with the intersection of the two Nash curves. We formulate this as a proposition and we then give a different proof for it using the IIA axiom.

Proposition 2. *The pair (w^N, L^N) that constitutes the Nash bargaining solution of the efficient bargaining problem (in the w–L space) coincides with the point of intersection of the two Nash curves.*

We do not make use of the fact that the two Nash curves intersect on the Pareto curve in the proof of Proposition 2. Rather, this is a result of the Nash solution satisfying the IIA axiom. This axiom prevents the intersection of the Pareto curve with either Nash curve being different from the solution of the efficient bargaining problem.

Figure 5 illustrates the efficient bargaining problem (S^e, d), as well as restricted ones for some values of L and w. The Nash solution $F(S^L(L^N), d)$ is only Pareto-efficient in the efficient bargaining problem (S^e, d) if L is fixed to L^N. In this case, the Nash solutions coincide. The same is true when we fix the wage at w^N and consider the restricted bargaining problem $(S^w(w^N), d)$. The curves denoted by $u(w^*)$ and $u(L^*)$ represent the parties' utilities along the Nash curves (i.e., they represent the set of Nash solutions for varying L and w, respectively). The two panels of Figure 5 are the counterparts to

Figure 5. Restricted bargaining problems for different values of w (left) and L (right)



Notes: The utility images of Figure 4 in which w or L are fixed at different levels. The utility image of the L-Nash curve (green) in the left panel is the collection of the NBS of the restricted bargaining problems with fixed w. The utility image of the w-Nash curve (blue) in the right panel is the collection with fixed L. The images of the Nash curves have a common intersection on the image of the Pareto curve (red) at the NBS.

Figure 4 in the utility space. As earlier, the arrows attached to the Nash curves indicate their "directions": along the arrows of the L-Nash curve, the wage level is increasing; along the w-Nash curve, employment is increasing.

6. Bargaining agendas

In the efficient bargaining problem, the two parties bargain over the two quantities simultaneously. Meanwhile, in the restricted problems, the wage or the employment level is only negotiated when the other quantity is treated as fixed. In this section, we investigate whether or not the bargaining problem over two quantities can be split into a sequence of alternating one-dimensional negotiations over the wage rate and the employment level. More precisely, we introduce two bargaining agendas that iteratively use restricted bargaining problems. Our question is whether the NBS of the efficient bargaining problem can be retrieved by iterated application of the NBS to restricted problems, and as a consequence the final agreement is Pareto-efficient.

Although our study of bargaining agendas should be understood as normative, we nonetheless observe a specific "order of negotiation" in particular negotiations. As an example, in the recent negotiations between *Deutsche Bahn* (the German Railway Company) and *Gewerkschaft der Lokführer* (the German Train Drivers' Union), there was an early settlement on the wage increase because *Deutsche Bahn's* offer matched the union's claim.[12] However, negotiations on working conditions and duration involved took another month. At the very end, the envisioned wage agreement was slightly adjusted. In addition to this example for a separation of issues, labour market institutions in many OECD countries (e.g., Germany or Austria) exhibit a separation connected with the coverage of agreements. Phrased in a stylized way, wages are negotiated on the sectoral level, while employment is later relegated to the company level (Du Caju et al., 2008). In that respect, the representatives of a party, who negotiate in the different rounds in the discussed agendas, need not be the same. It suffices to think of them as having identical preferences.

6.1. Two-stage bargaining agenda

In this agenda, we apply the two restricted bargaining problems in which the parties negotiate one of the two quantities separately, as discussed in Section 4, to model a sequential two-stage process of one-dimensional bargaining problems. In each stage, one of these restricted problems has to be negotiated. The agreement in Stage 1 fixes either quantity and thus shapes the bargaining problem at Stage 2 over the other quantity. Depending on the order in which the wage and employment are negotiated, we obtain two versions of the agenda, as follows.

Agenda Ia (first wage, then employment). In Stage 1, the players agree on a wage, w, and in Stage 2 they agree on an employment level, L. The final outcome is the agreed pair (w, L).

Agenda Ib (first employment, then wage). In Stage 1, the players agree on a wage, L, and in Stage 2 they agree on an employment level, w. The final outcome is the agreed pair (w, L).

A solution concept for this two-stage bargaining problem is certainly connected to the bargaining solution that is used for a single bargaining problem. For the analysis, we employ a cooperative backward induction approach, which means that it is common knowledge that any bargaining problem is solved using the same commonly accepted bargaining solution, and therefore the outcome of any subsequent bargaining problem can be anticipated. Following our discussion in the previous sections, we restrict our attention to the Nash bargaining solution here; in Section 7, we briefly discuss

[12] See https://www.zeit.de/wirtschaft/unternehmen/2021-08/deutsche-bahn-gdl-tarifverhandlungen-bahnchef-angebot-entgegenkommen-verhandlungstisch (in German).

the differences when other solution concepts (e.g., the Kalai–Smorodinsky solution) are used instead.

Consider Agenda Ia under the assumption that the Nash bargaining solution is applied throughout. Assume further that parties have negotiated a wage level and agreed on $\tilde{w}$ in Stage 1. Then, for the negotiations over L at Stage 2, the set of feasible utility allocations is given by $S^w(\tilde{w})$. Anticipating that the Nash solution will be applied at Stage 2, the final agreement will be $(\tilde{w}, L^*(\tilde{w}))$ (i.e., it is located on the L-Nash curve). In other words, when following Agenda Ia, the parties effectively negotiate all (w, L) combinations on the L-Nash curve. Analogously, the bargaining problem at Stage 1 over L in Agenda Ib effectively describes negotiations over (w, L) combinations on the w-Nash curve.

Proposition 3. *Assume that, in Agenda Ia and Agenda Ib, players apply the Nash bargaining solution at Stage 2. Then, there is a well-defined bargaining problem in Stage 1. Application of the Nash solution in Stage 1 yields a solution on the Pareto curve. This solution coincides with the Nash solution of the efficient bargaining problem.*

Another way to look at the results in the two-stage agendas is to inspect Figure 4 and the iso-product lines (i.e., the iso-level curves of the Nash product). In Agenda Ia, the final agreement on wage is reached by selecting the point on the L-Nash curve that maximizes the product of utilities. As depicted in Figure 4, the maximizer (w, L) is the intersection point of the Pareto curve and the L-Nash curve. The IIA axiom guarantees that disentangling the simultaneous negotiation over the two quantities into a two-stage game still leads to the Nash solution of the efficient bargaining problem. This happens because the Nash solution of the efficient bargaining problem is feasible in the Stage 1 bargaining problem (either in Ia or Ib).

6.2. Iterated bargaining agenda

The results in Propositions 3 come at the cost that the determination of the Stage 1 bargaining problem requires the calculation of the Nash solution for any possible $S^w(w)$ or $S^L(L)$. To reduce this type of computational effort, we introduce a second agenda in two versions according to which the parties negotiate with infinite horizon, alternating the quantity over which they bargain. Again, we assume that all bargaining problems are solved by using the Nash bargaining solution.

Agenda IIa (iterated separate bargaining *w*-start). Players alternately bargain over w and L, starting with negotiations over the wage level w.

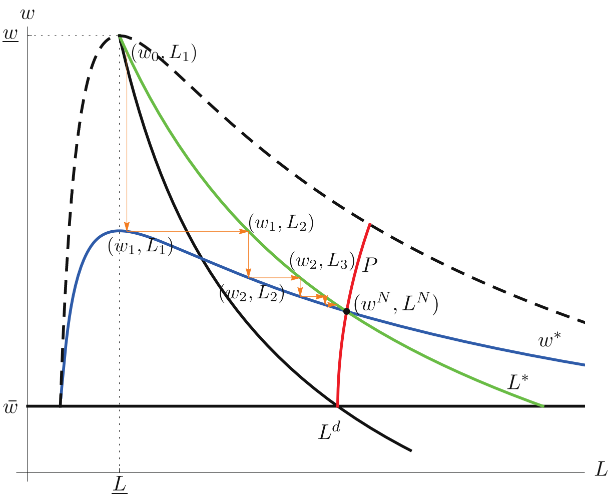
Agenda IIb (iterated separate bargaining L-start). Players alternately bargain over w and L, starting with negotiations over the employment level L.

To illustrate the idea behind Agenda IIa, suppose that each party sends two agents to form two distinct negotiation teams in two distinct rooms. In the w-room, the parties' agents only negotiate a wage, while in the L-room, only L is negotiated. To start negotiations, fix $w_0 \in \mathrm{Int}(\mathcal{W})$ and announce it to the L-room. Given w_0, there is an agreement on L_1 in the L-room that is reported to the w-room. Given L_1, parties negotiate a wage and agree on w_1. This is reported to the L-room, and so on. Therefore at any iteration of the process a single restricted bargaining problem needs to be solved. The questions are whether the process converges and, if so, then to what limit.

Proposition 4. *Assuming that negotiations are resolved with the Nash bargaining solution, the sequence of intermediate solutions converges to the Nash bargaining outcome of the efficient bargaining problem in both Agendas IIa and IIb.*

Figure 6 illustrates the process with starting wage w_0 just below $\underline{w}$. The negotiation result L_1 in the L-room is such that (w_0, L_1) is located on the L-Nash curve. With fixed L_1, the Nash solution wage of the restricted problem $S^L(L_1)$ is w_1, so that (w_1, L_1) lies on the w-Nash curve. In general, for $t \in \mathbb{N}$

Figure 6. Illustration of the iteration process of Agenda IIa



Notes: The arrows (orange) represent the dynamics in Agenda IIa. Starting with fixed w_0, the employment and the wage rate are alternately negotiated, which corresponds to switching between the L- and the w-Nash curve towards the limit, which is the Nash bargaining solution of the efficient bargaining problem.

the points (w_{t-1}, L_t) are located on the L-Nash curve, while (w_t, L_t) is on the w-Nash curve. The relative position of the two Nash curves as shown in Proposition 1 guarantees the convergence of the sequence to the Nash solution of the efficient bargaining problem.

7. Other bargaining solutions

We are interested in how far our results depend on the choice of the Nash bargaining solution. To this end, we briefly review three alternative solution concepts: the egalitarian solution (Kalai, 1977), the utilitarian solution (Thomson, 1981), and the Kalai–Smorodinsky solution (Kalai and Smorodinsky, 1975). It is well known that the former two satisfy the IIA axiom, while the latter does not.

7.1. Egalitarian and utilitarian solutions

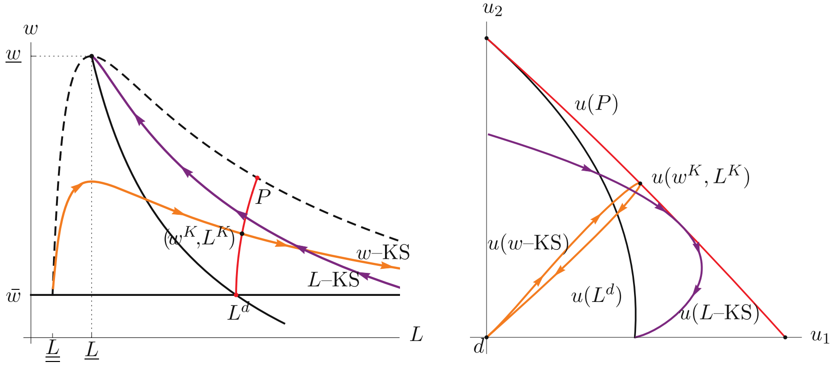
For a bargaining problem (S, d), the egalitarian solution F^E selects the weakly Pareto efficient point $F^E(S, d)$ in S, for which $F_1^E(S, d) - d_1 = F_2^E(S, d) - d_2$ holds. The utilitarian solution F^U determines the point $F^U(S, d)$ at which the sum of excess utilities (above d) is maximized. While the egalitarian solution can lead to a weakly Pareto optimal point, the utilitarian solution does not necessarily produce an individually rational result. However, both solutions satisfy the IIA axiom, so that the solution curves for the two one-dimensional bargaining problems intersect on the Pareto curve for either solution concept. Thereby, given that the utilitarian solution point is independent of the status quo point, its two solution curves in the w–L space are independent of exogenous quantities, such as $\overline{w}$, N, or C. It thus follows that in analogy to our findings on the Nash solution, the repeated bargaining agenda eventually leads to the solution of the efficient bargaining problem. More generally, any bargaining solution that is defined as the maximizer of a monotonic, quasi-concave function over the individual rational utility allocations satisfies Pareto efficiency, individual rationality, and the IIA axiom.[13] Consequently, the corresponding two solution curves have a common intersection on the Pareto curve, which is the solution of the efficient bargaining problem.

7.2. The Kalai–Smorodinsky solution

Besides the Nash bargaining solution, the solution introduced by Kalai and Smorodinsky (1975) is the second most applied solution in the labour market

[13] Monotonicity of the function guarantees Pareto efficiency, individual rationality is trivial, when only individual rational points are under consideration, and IIA results from the fact that a maximizer $\overline{x}$ is also a maximizer over any subset that contains $\overline{x}$.

Figure 7. The KS curves in the w–L space (left) and the utility space (right)



Notes: The solution curves for the KS solution: the L-KS curve (purple) and the w-KS curve (orange) do not intersect on the Pareto curve (red).

literature (see the references cited in the final paragraph of Section 2). To determine the Kalai–Smorodinsky (KS) solution, we first calculate the maximal possible utilities among all of the individually rational utility allocations. The KS solution selects the Pareto-efficient point in S where each player receives the same share of their maximal possible excess utility. The original axiomatization of the KS solution rests on the axiomatization of the Nash solution and replacing the IIA axiom by the axiom of individual monotonicity.[14] We discuss the lack of IIA along Figure 7, which illustrates the two KS curves in the w–L space (left panel) and also their images in utility space (right panel). For better comparability, we use the same specification of the model as used in the previous sections and as in Figures 4 and 5.

The construction of the L-KS and w-KS curves follows the same route as with the Nash curves: for any fixed w, the corresponding point on the L-KS curve marks the employment level L, so that the utility allocation in (w, L) coincides with the KS solution of the restricted bargaining problem $S^w(w)$, and analogously for the w-KS curve. The left panel of Figure 7 shows that the relative locations of the L-KS and w-KS curves are the same as in Proposition 1 for the Nash curves. However, although the two KS curves are qualitatively similar to the Nash curves, the main difference is that the intersection point of the two KS curves in the w–L space is not located on the Pareto curve. Moreover, the utility allocation of the KS solution of the efficient bargaining

[14]In experimental studies on unstructured bargaining over two and three alternatives, Galeotti et al. (2019, 2022) provide some evidence that the IIA axiom might not be satisfied (in a probabilistic sense) confirming criticism of the IIA axiom in the literature that, for example, motivated the solution of Kalai and Smorodinsky (1975).

problem, $u(w^K, L^K)$, is not located on the image of either the w-KS or the L-KS curve. In other words, even if the wage (resp. employment) level were fixed to w^K (resp. L^K), the negotiated employment level L (resp. wage level w) in the restricted bargaining problem $S^w(w^K)$ (resp. $S^L(L^K)$) will not be equal to L^K (resp. w^K), so that the outcome is inefficient.[15] The immediate consequence is that Propositions 3 and 4 do not hold for the KS solution. In particular, Agendas Ia and Ib typically lead to two different inefficient outcomes on the L-KS curve and w-KS curve, respectively. The final outcome of the iterated bargaining agenda is the intersection point of the two KS curves, and is therefore not efficient.

The right panel of Figure 7 displays the situation in the utility space. There are two intersection points of the images of KS curves in the utility space. The left-hand intersection point stems from the intersection of the two curves in the w–L space, while the right-hand intersection point is the image of two distinct (w, L) combinations yielding the same utilities but one is located on the L-KS curve and the other is located on the w-KS curve. In particular, the former intersection point is the result of the repeated bargaining agenda in utilities and is apparently not Pareto-efficient. These considerations again highlight the fundamental significance of the IIA axiom for the results in Propositions 1–4.

8. Conclusion

We present a rigorous analysis of the two-dimensional bargaining model on the labour market introduced by McDonald and Solow (1981). We formulate this bargaining problem not only within the space of physical outcomes (w, L) but also in the utility space, so that an application of axiomatic bargaining solutions such as the NBS is possible. A decomposition of the two-dimensional bargaining problem over the wage rate and the employment level into two (families of) one-dimensional bargaining problems results in the construction of two curves, which we refer to as the w-Nash curve and the L-Nash curve. While the former is disregarded in the literature, the prevalent interpretation of the latter is merely technical as parametrized solutions of a first-order condition. However, both of these curves are equipped with an economic interpretation: each collects outcomes of the Nash bargaining solution applied to restricted (one-dimensional) problems or represents the generalized bargaining solution of a parametrized one-dimensional bargaining problem.

[15] Although it seems that the solution (w^K, L^K) does lie on the w-KS curve in Figure 7, this is not true; (w^K, L^K) only lies on the w-KS curve if the Pareto curve is vertical. (This is also confirmed by numerical calculations.)

A stringent analysis of the curves characterizes their (relative) positions in the w–L space. The unique intersection point satisfies stability in the sense that either quantity at the intersection point marks the NBS outcome of the restricted bargaining problem, in which the other quantity is fixed to the level at the intersection, and vice versa. The strength of the IIA axiom (and not just bare calculus) is that it forces the intersection point to coincide with the NBS outcome of the efficient bargaining problem.

Knowledge of the structural properties of the two Nash curves leads us to design two dynamic bargaining agendas, according to which either the wage rate or the employment level is negotiated at each stage. Independent of whether each quantity is negotiated by prescient parties once, or quantities are negotiated alternately by myopic agents (with infinite horizon), the final outcome coincides with the intersection point of the two Nash curves – and, as a consequence, is Pareto-efficient. Again, the driving force for these results is the IIA axiom, which constitutes the special features of the NBS.

Although our analysis heavily relies on the NBS, our approach is not limited to this solution concept. Any axiomatic bargaining solution, as is usually defined in utility space, can be transferred to the w–L space (i) to mark the outcome of the solution concept, and (ii) to construct two solution curves with the corresponding interpretation as used for the NBS earlier. However, the set of axioms of a bargaining solution determines to what extent our results hold for a specific solution concept. In particular, the KS solution fails to satisfy the IIA axiom, and therefore the common intersection point of the two KS curves is not Pareto-efficient and does not coincide with the KS solution of the efficient bargaining problem.

Our analysis of sequential bargaining agendas is rather normative but can also be interpreted as a policy advice when designing protocols for collective bargaining problems. Sequential negotiations do not necessarily lead to an inefficient or unbalanced outcome, per se. Actually, from a practical point of view, our limited result for infinitely alternating negotiations means that intermediate agreements do not change significantly after finitely many (and possibly few) rounds. Therefore, alternatingly ignoring parts of the scope might facilitate the agreement without loss of efficiency or fairness.

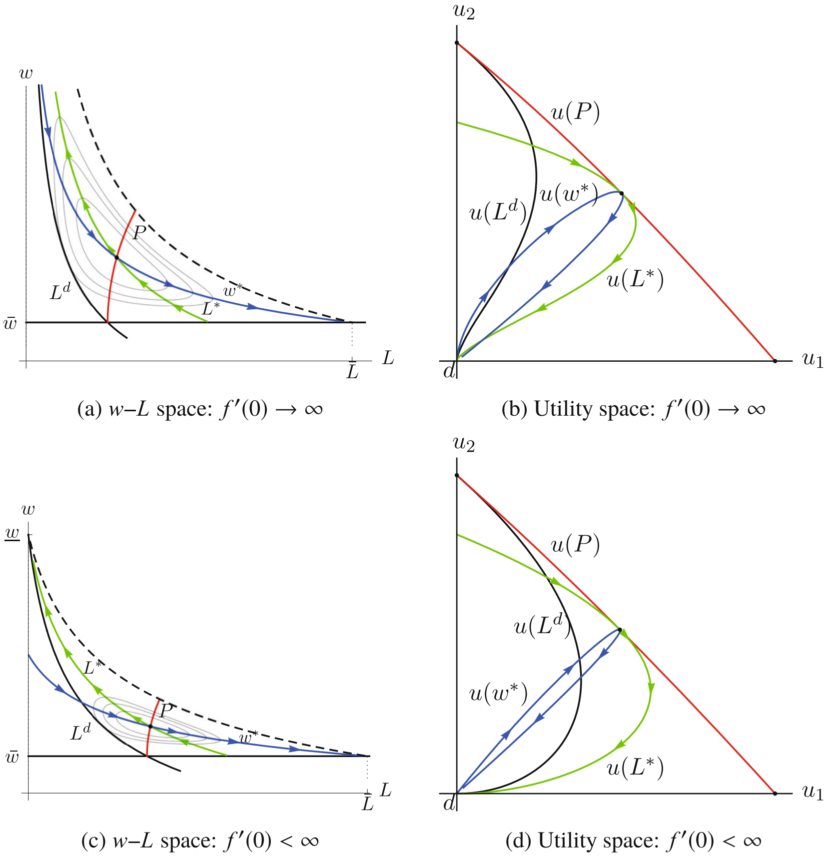
We consider the following three points as the main insights from our theoretical analysis. First, the IIA axiom implants more structure into the NBS than was uncovered in the literature; this structure enables us to safely decompose the wage bargaining problem into families of one-dimensional bargaining problems preserving the solution. Second, one has to be careful when applying a particular bargaining solution because the varying characterizing axioms possibly trigger unwanted results. What leads to an efficient outcome for one solution concept can cause inefficiencies for another. Finally, our analysis gives rise to further investigation of the NBS in other application contexts. The decomposability does not appear to hinge on the fact that parties bargain over two quantities. Instead, when more than two different quantities are subject to negotiation (e.g., when the buyer and seller agree on different terms of trade), a decomposition into restricted bargaining problems over one or simply fewer dimensions seems to be possible and computationally less demanding. To what degree our findings hold in such more general models is an interesting open question.

Appendix A. Zero set-up cost

We drop the assumption that the firm has to incur set-up costs for production, and set $C = 0$, so that the firm's profit function reduces to $u_2(w, L) = f(L) - wL$. We then need to distinguish two cases whether f satisfies an Inada condition or not (i.e., whether $f'(0)$ is infinite or finite). Figure A1 illustrates the analogous versions of Figures 1 and 5 for the cases $f'(0) \to \infty$ and $f'(0) < \infty$. Given that removing set-up costs is accomplished via a monotone transformation of the profit function, the shapes of iso-profit lines do not change, and therefore the Pareto curve is not affected by setting $C = 0$. It is straightforward that here $\underline{L} = 0$ and that $\underline{w}$ is no longer finite. Consequently, the L-Nash curve, the zero profit line and the labour demand curve go to infinity, when L approaches $\underline{L} = 0$. From Section 5.2, we know that $w^*(L) = w^0(L)\sigma(w^*(L))/(1 + \sigma(w^*(L))) < w^0(L)$ for any employment level L, so that as L approaches 0, the w-Nash curve also tends to infinity. As we see from Figure A1(a), the result obtained in the previous sections is still valid: the intersection of the two Nash curves lies on the Pareto curve, and thus represents the Nash solution of efficient bargaining problem. The argumentation via the IIA axiom remains valid.

Figure A1(c) represents the case in which the marginal product at $L = 0$ is finite. The zero profit line and labour demand curve have a common intersection on the wage axis at $\underline{w} = f'(0)$. In addition, the L-Nash curve intersects with these two curves at $\underline{w}$. In the case of the w-Nash curve, when L goes to $\underline{L} = 0$, the w-Nash curve approaches $(0, w)$ with a wage level between $\overline{w}$ and $\underline{w}$ as before. The two Nash curves have exactly one common point, which lies on the Pareto curve and coincides with the Nash solution of the efficient bargaining problem. Hence, all of our results obtained in Sections 3–6 continue to hold in the case of vanishing set-up costs. In both cases (i.e., for a finite or an infinite marginal product $f'(0)$), we can retrieve the results from

Figure A1. Adaptation of Figures 4 and 5 to the case of zero set-up costs



Notes: In panels (a) and (b), the Inada condition holds; in panels (c) and (d), it does not. When $f'(0) \to \infty$, then $\underline{L} \to 0$; else $\underline{L} = 0$.

Lemma 5, Corollary 1, and Proposition 1 on monotonicity of the Nash curves and their relative position. As a result, Proposition 4 holds so that the final outcome in the iterated bargaining Agendas IIa and IIb is the Nash bargaining outcome.

Finally, the results for Agendas Ia and Ib (i.e., Proposition 3) remain valid. This can be verified in Figures A1(b) and (d). The bargaining problems in Agendas Ia and Ib are formed by the L-Nash curve and the w-Nash curve, respectively. By the IIA axiom, the Nash solution of the efficient bargaining problem coincides with the Nash solution in each Stage 1 bargaining problem, which is assumed to be the final outcome in both agendas. We thus conclude that the assumption of positive set-up costs in previous sections is convenient because it does not necessitate explicit consideration of an Inada condition; yet, our results are independent of this assumption.

Appendix B. Proofs

Proof of Lemma 1: Differentiation of σ yields

$$\sigma'(w) = \frac{(v(w) - \overline{v})\,(wv''(w) + v'(w)) - (v'(w))^2 w}{(v(w) - \overline{v})^2} = \frac{v'(w)\,(\varepsilon(w) - \sigma(w) + 1)}{v(w) - \overline{v}}, \tag{B1}$$

from which the result follows immediately, as $v' > 0$ and $w > \overline{w}$ by hypothesis. □

Proof of Lemma 2: Because $(w - f'(L))/w < 1$, it follows from equation (5) that for any Pareto-efficient allocation (w, L) we must have $\sigma(w) > 1$. □

Proof of Lemma 3: Straightforward calculations show

$$P'(L) = \frac{dw}{dL}\Big|_{\Phi^P = 0} = -\frac{\partial \Phi^P(w, L)/\partial L}{\partial \Phi^P(w, L)/\partial w} = \frac{wf''(L)}{\sigma'(w)(w - f'(L))^2 + f'(L)} = \frac{\sigma(w)f''(L)}{\varepsilon(w)} > 0,$$

where in the last equation we used the definition of σ along with equations (B1) and (5). Hence, P has a positive slope, unless it is vertical, which happens if $v'' = 0$. Finally,

$$\lim_{w \searrow \overline{w}} \sigma(w) = +\infty \quad \Rightarrow \quad \lim_{w \searrow \overline{w}} \frac{\sigma(w)f''(L)}{\varepsilon(w)} = \frac{f''(L^c)}{\varepsilon(\overline{w})} \lim_{w \searrow \overline{w}} \sigma(w) = +\infty,$$

which shows that the slope of the Pareto curve is infinite at $(w^c, L^c) = (\overline{w}, L^d(\overline{w}))$. □

Proof of Lemma 4:

(i) (S^L, d) *is well-defined for fixed* L. Comprehensiveness of S^L follows from the definition. To show convexity, we show that the slope of the Pareto boundary of S^L decreases with increasing utility of the trade union. The derivatives,

$$\frac{\partial u_1(w, L)}{\partial w} = Lv'(w) > 0, \qquad \frac{\partial u_2(w, L)}{\partial w} = -L < 0,$$

imply that for each w the utility allocation $u(w, L)$ is strictly Pareto optimal. The slope of the Pareto boundary at $u(w, L)$ is given by

$$\frac{du_2}{du_1} = \frac{\partial u_2(w, L)/\partial w}{\partial u_1(w, L)/\partial w} = \frac{-1}{v'(w)} < 0,$$

which is decreasing in w, as $v'' < 0$. Therefore, the function describing the strict Pareto boundary of S^L is concave, hence S^L is convex.
The closedness of S^L is a consequence of the continuity of the utility functions. Boundedness follows from the fact that $u_2(w, L)$ is strictly decreasing in w with $\lim_{w \to \underline{w}} u_2(w, L) \leq 0$. Finally, the disagreement point $d = (N\overline{v}, 0)$ is Pareto-dominated by $u(\overline{w}, L) \in S$ because $u_1(\overline{w}, L) = N\overline{v}$ and $u_2(\overline{w}, L) \geq 0$. With comprehensiveness, $d \in S^L$.

(ii) *(S^w, d) is well-defined for fixed w.* S^w is closed, comprehensive and the individually rational set is bounded with similar arguments as above.

$$\frac{\partial u_1(w, L)}{\partial L} = v(w) - \overline{v} > 0, \qquad \frac{\partial u_2(w, L)}{\partial L} = f'(L) - w,$$

mean that for $L < f'^{-1}(w) = L^d(w)$ the payoff allocation $u(w, L)$ is not Pareto-efficient because an increase in L increases both parties' utilities. Hence, the Pareto boundary of S^w is generated by all L such that $f'(L) < w$. Its slope at $u(w, L)$ is

$$\frac{du_2}{du_1} = \frac{\partial u_2(w, L)/\partial L}{\partial u_1(w, L)/\partial L} = \frac{f'(L) - w}{v(w) - \overline{v}},$$

which is negative for $L > L^d(w)$. Differentiating the last equation with respect to L yields $f''(L)/(v(w) - \overline{v}) < 0$, showing the convexity of S^w. Any wage $w > \overline{w}$ guarantees a non-negative utility to the workers, while for $w \leq \underline{w}$, the firm obtains a non-negative profit for all $L \geq \underline{L}$ with $f'(L) = w$. Therefore, the disagreement point d is Pareto-dominated and, by comprehensiveness from the definition, it belongs to S^w. □

Proof of Lemma 5: Let $M(L)$ denote the maximand in equation (8). We first show that for each fixed $w > \overline{w}$, $M(L)$ is strictly quasi-concave, and therefore admits a unique maximizer. It is sufficient to show strict quasi-concavity of the function Π with

$$\Pi(L) := L(f(L) - w(L) - C) = Lu_2(w, L),$$

because M and Π only differ in the constant factor $v(w) - \overline{v}$.

Next we show that for $L_1, L_2 \in \mathcal{L}$ with $L_1 \neq L_2$ and $\lambda \in (0, 1)$ we have $\Pi(L_\lambda) > \Pi_{min} := \min(\Pi(L_1), \Pi(L_2))$, where $L_\lambda := \lambda L_1 + (1 - \lambda)L_2$. Due to concavity of f, we obtain

$$\begin{aligned}
\Pi(L_\lambda) &= L_\lambda(f(L_\lambda) - wL_\lambda - C) \\
&\geq L_\lambda(\lambda f(L_1) + (1-\lambda) f(L_2) - wL_\lambda - C) \\
&= L_\lambda(\lambda u_2(w, L_1) + (1-\lambda) u_2(w, L_2)) \\
&= (\lambda L_1 + (1-\lambda) L_2)(\lambda u_2(w, L_1) + (1-\lambda) u_2(w, L_2)) \\
&= \lambda^2 \Pi(L_1) + (1-\lambda)^2 \Pi(L_2) + \lambda(1-\lambda)\left(L_1 u_2(w, L_2) + L_2 u_2(w, L_1)\right) \\
&= \left(\lambda^2 + \lambda(1-\lambda)\frac{L_2}{L_1}\right)\Pi(L_1) + \left((1-\lambda)^2 + \lambda(1-\lambda)\frac{L_1}{L_2}\right)\Pi(L_2) \\
&\geq \left(\lambda^2 + \lambda(1-\lambda)\frac{L_2}{L_1}\right)\Pi_{min} + \left((1-\lambda)^2 + \lambda(1-\lambda)\frac{L_1}{L_2}\right)\Pi_{min} \\
&= \left(\lambda^2 + \lambda(1-\lambda)\frac{L_2}{L_1} + (1-\lambda)^2 + \lambda(1-\lambda)\frac{L_1}{L_2}\right)\Pi_{min} \\
&= \left(1 + (\lambda - \lambda^2)\frac{(L_1 - L_2)^2}{L_1 L_2}\right)\Pi_{min} > \Pi_{min}.
\end{aligned}$$

Thus, Π is a strictly quasi-concave function. This implies the quasi-concavity of M, so that M has a unique maximizer. Inserting the specific forms of u_1, u_2, d_1, and d_2, the first-order condition $M'(L) = 0$ is thus sufficient to have the functional form of the L-Nash curve. $M'(L) = (v(w) - \overline{v})(f(L) - 2wL - C + Lf'(L)) = 0$ implies

$$w = \frac{1}{2}\left(\frac{f(L) - C}{L} + f'(L)\right).$$

To show that the L-Nash curve is downward sloping, we need $w'(L) < 0$:

$$w'(L) = \frac{1}{2L^2}\left(\underbrace{L^2 f''(L)}_{<0} - \underbrace{[f(L) - f'(L)L - C]}_{\geq 0 \text{ for } L \geq \underline{L}}\right) < 0.$$

Finally, if $\underline{w}$ is finite, then as w converges to $\underline{w}$ the individually rational part of $S^w(w)$ shrinks to the disagreement point. By definition, $\underline{L}$ is the employment level, and so $u(\underline{w}, \underline{L}) = d$. Hence, $(\underline{w}, \underline{L})$ is the starting point of the L-Nash curve. □

Proof of Lemma 6: We first show that the maximand in equation (10) is strictly concave, yielding a unique maximizer. Inserting the specific forms of

u_1, u_2, d_1, and d_2, we view the product as a function M in w:

$$(u_1(w,L) - d_1)\ (u_2(w,L) - d_2) = L(v(w) - \overline{v})(f(L) - wL - C) =: M(w),$$

with second derivative $M''(w) = Lv''(w)u_2(w,L) - 2L^2v'(w) < 0$. The first-order condition $M'(w) = 0$ is thus sufficient and readily reads $\sigma(w) - [wL/u_2(w,L)] = 0$, so that the w-Nash curve is implicitly defined. Using the implicit function theorem, the slope of the w-Nash curve is given by

$$\begin{aligned}\frac{dw^*(L)}{dL} &= \frac{dw}{dL}\Big|_{\Phi^w(w,L)=0} = -\frac{\partial\Phi^w(w,L)/\partial L}{\partial\Phi^w(w,L)/\partial w}\\ &= \frac{w\,(f(L) - C - Lf'(L))}{\sigma'(w)(f(L) - Lw - C)^2 - L(f(L) - C)}.\end{aligned}$$

Due to Assumption 1, the numerator is positive. The denominator is negative if $\sigma'(w) \leq 0$ holds. Thus, the w-Nash curve is downward-sloping in the neighbourhood of the Pareto curve. Finally, $w^*(L)$ at fixed L has to be located between $\overline{w}$ (zero excess utility for the trade union) and $(f(L) - C)/L$ (zero profit wage). By definition of $\overline{L}$, $\lim_{L\to\overline{L}} w^0(L) = \overline{w}$. □

Proof of Proposition 1: We start with two considerations on the relative positions of starting points of the two curves. By Lemma 5, the L-Nash curve is located between the labour demand curve and the zero profit line, which have a common point in $(\underline{w}, \underline{L})$. Therefore, as L approaches $\underline{L}$, the corresponding points on the L-Nash curve yield arbitrarily small profits for the firm. However, along the w-Nash curve, profits cannot approach zero for L close to $\underline{L}$ because the wage w is chosen so as to maximize the product of the parties' excess utilities. Consequently, the w-Nash curve must start below the L-Nash curve at $L = \underline{L}$.

Analogously, for w approaching $\overline{w}$, the corresponding L is such that (w, L) on the w-curve approaches $\overline{L}$. Recall that both parties utilities are zero in $(\overline{w}, \overline{L})$. It follows that the endpoint of the L-Nash curve at $\overline{w}$ is given at some employment level $L \leq \overline{L}$. These observations on the starting points and the endpoints together with the monotonicity of both curves (see Lemmas 1 and 5) show that there must be at least one intersection point of the two curves.

Inserting the closed form for the L-Nash curve (9) into equation (11) immediately yields the condition for Pareto-efficient agreements (5). This shows that any intersection point must be located on the Pareto curve. By Lemma 3, the Pareto curve is upward-sloping, which shows that it intersects each Nash curve at most once. Hence, the two Nash curves have a unique intersection, which is located on the Pareto curve. Again, using the location of starting points and endpoints shows (iii). □

Proof of Proposition 2: Because the Nash solution is Pareto-efficient, (w^N, L^N) must be located on the Pareto curve. As noted above, comparing the efficient bargaining problem with the restricted ones, in which wage or employment is fixed to w^N or L^N, respectively, gives $S^w(w^N) \subseteq S^e$ and $S^L(L^N) \subseteq S^e$. (w^N, L^N) is among the possible agreements from which the bargaining problems $(S^L(L^N), d)$ and $(S^w(w^N), d)$, respectively, are generated. Therefore, $F(S^e, d) = u(w^N, L^N)$ is included in $S^L(L^N)$ and $S^w(w^N)$. Using $S^w(w^N) \subseteq S^e$, $S^L(L^N) \subseteq S^e$ and the IIA axiom for the Nash solution, we obtain $F(S^L(L^N), d) = F(S^e, d^e) = u(w^N, L^N)$ and $F(S^w(w^N), d) = F(S^e, d^e) = u(w^N, L^N)$. It follows by definition of the Nash curves that the pair (w^N, L^N) must be located on both Nash curves. □

Proof of Proposition 3: By the construction of two-stage bargaining, the bargaining problem at the first stage is (S, d) with $S = \{x \in \mathbb{R}^2 \mid x \leq (u_1(w, L^*(w)), u_2(w, L^*(w))), w \in \mathcal{W}\}$ and $d = (N\bar{v}, 0)$, which is well defined by Lemma 4. Because the Nash solution is Pareto-efficient, the solution point $(w, L^*(w))$ is on the Pareto curve. By the application of the Nash solution in the first stage along with IIA, we have $u(w, L^*(w)) = F(S, d) = F(S^e, d) = u(w^N, L^N)$. Thus, $(w, L^*(w))$ and (w^N, L^N) coincide.

An adaptation of the proof to Agenda Ib is straightforward, so that we omit it. Which variable the negotiation starts with does not have an effect on the result (i.e., the result is independent of the order of negotiation). □

Proof of Proposition 4: We show the proposition for Agenda IIa. Fix $w_0 \in \mathrm{Int}(\mathcal{W})$. Let L_1 be such that $a^1 := (w_0, L_1)$ satisfies equation (9); that is, (w_0, L_1) is located on the L-Nash curve. Then compute w_1 such that $a^2 := (w_1, L_1)$ satisfies equation (11); that is, (w_1, L_1) is located on the w-Nash curve. Proceeding in this way, we construct sequences $(w_t)_t, (L_t)_t$ of wages and employment levels, as well as the sequence $(a^k)_k$ of wage/employment combinations such that $a^k = (w_{k/2}, L_{k/2})$, when k is even and $a^k = (w_{(k-1)/2}, L_{(k+1)/2})$ when k is odd. This means that for even k, a^k is located on the w-Nash curve, while a^k lies on the L-Nash curve for odd k. Using Proposition 1, the sequences $(w_t)_t$ and $(L_t)_t$ are monotonic (increasing or decreasing, depending on w_0) and are bounded by w^N and L^N, respectively. It follows that both converge to, say, $\tilde{w}$ and $\tilde{L}$, respectively. Then, a^k is a Cauchy sequence and $(a^k - a^{k+1})_k$ converges to $(\tilde{w}, \tilde{L}) - (\tilde{w}, \tilde{L}) = 0$, which implies that $(\tilde{w}, \tilde{L})$ must be in the intersection of the L-Nash curve and the w-Nash curve. Thus, $(\tilde{w}, \tilde{L}) = (w^N, L^N)$.

The proof for Agenda IIb is immediate. One can view the negotiated wage level of the first round as fixed and can apply the above arguments. □

References

Aidt, T. S. and Tzannatos, Z. (2008), Trade unions, collective bargaining and macroeconomic performance: a review, *Industrial Relations Journal 39*, 258–295.

Albrecht, J. and Vromen, S. (2002), A match model with endogenous skill requirements, *International Economic Review 43*, 283–305.

Alexander, C. (1992), The Kalai–Smorodinsky bargaining solution in wage negotiations, *Journal of the Operational Research Society 43*, 779–786.

Alexander, C. O. and Ledermann, W. (1994), The constrained Nash bargaining solution, *Journal of the Operational Research Society 45*, 954–958.

Alexander, C. O. and Ledermann, W. (1996), Are Nash bargaining wage agreements unique? An investigation into bargaining sets for firm–union negotiations, *Oxford Economic Papers 48*, 232–253.

Amine, S., Lages dos Santos, P., Baumann, S., and Valognes, F. (2009), Revisiting Nash wages negotiations in matching models, *Economics Bulletin 29*, 3203–3213.

Amine, S., Baumann, S., and Dos Santos, P. L. (2018), Bargaining solutions and public policies in matching models, *Economic Studies Journal 27*, 3–14.

Bayındır-Upmann, T. and Raith, M. G. (2003), Should high-tax countries pursue revenue-neutral ecological tax reforms?, *European Economic Review 47*, 41–60.

Bayındır-Upmann, T. and Raith, M. G. (2005), Unemployment and pollution: is one policy suited for two problems?, *The Economic Record 81*, 378–393.

Belan, P., Carre, M., and Gregoir, S. (2010), Subsidizing low-skilled jobs in a dual labor market, *Labour Economics 17*, 776–788.

Binmore, K., Rubinstein, A., and Wolinsky, A. (1986), The Nash bargaining solution in economic modelling, *Rand Journal of Economics 17*, 176–188.

Boeri, T. and Burda, M. C. (2009), Preferences for collective versus individualised wage setting, *Economic Journal 119*, 1440–1463.

Bughin, J. (1996), Trade unions and firms' product market power, *Journal of Industrial Economics 44*, 289–307.

Bughin, J. (1999), The strategic choice of union–oligopoly bargaining agenda, *International Journal of Industrial Organization 17*, 1029–1040.

Cahuc, P., Carcillo, S., and Zylberberg, A. (2014), *Labor Economics*, 2nd edn, MIT Press, Cambridge, MA.

Clark, A. (1990), Efficient bargains and the McDonald–Solow conjecture, *Journal of Labor Economics 8*, 502–528.

Colciago, A. and Rossi, L. (2015), Firm dynamics, endogenous markups, and the labor share of income, *Macroeconomic Dynamics 19*, 1309–1331.

Creane, A. and Davidson, C. (2011), The trade-offs from pattern bargaining with uncertain production costs, *European Economic Review 55*, 246–262.

Creedy, J. and McDonald, I. M. (1991), Models of trade union behaviour: a synthesis, *The Economic Record 67*, 346–359.

Dittrich, M. (2010), Minimum wages and unemployment benefits in a unionised economy: a game-theoretic approach, *Annals of Economics and Finance 11*, 209–229.

Dittrich, M. and Knabe, A. (2013) Spillover effects of minimum wages under union wage bargaining, *Journal of Institutional and Theoretical Economics 169*, 506–518.

Dobson, P. W. (1994), Multifirm unions and the incentive to adopt pattern bargaining in oligopoly, *European Economic Review 38*, 87–100.

Dowrick, S. (1989), Union–oligopoly bargaining, *Economic Journal 99*, 1123–1142.

Dowrick, S. (1990), The relative profitability of Nash bargaining on the labour demand curve or the contract curve, *Economics Letters 33*, 121–125.

Du Caju, P., Gautier, E., Momferatou, D., and Ward-Warmedinger, M. (2008), Institutional features of wage bargaining in 23 European countries, the US and Japan, IZA Discussion Paper 3867.

Dutt, A. K. and Sen, A. (1997), Union bargaining power, employment and output in a model of monopolistic competition with wage bargaining, *Journal of Economics (Zeitschrift fur Nationalokonomie) 65*, 1–17.

Eichner, T. and Upmann, T. (2012), Labour markets and capital tax competition, *International Tax and Public Finance 19*, 203–215.

Eichner, T. and Upmann, T. (2014), The (im)possibility of overprovision of public goods in interjurisdicitional tax competition, *FinanzArchiv/ Public Finance Analysis 70*, 218–248.

Fanti, L. and Gori, L. (2013), Efficient bargaining versus right to manage: a stability analysis in a Cournot duopoly with trade unions, *Economic Modelling 30*, 205–211.

Flinn, C. J. (2006), Minimum wage effects on labor market outcomes under search, matching, and endogenous contact rates, *Econometrica 74*, 1013–1062.

Galeotti, F., Montero, M., and Poulsen, A. (2019), Efficiency versus equality in bargaining, *Journal of the European Economic Association 17*, 1941–1970.

Galeotti, F., Montero, M., and Poulsen, A. (2022), The attraction and compromise effects in bargaining: experimental evidence, *Management Science 68*, 2377–3174.

Garnero, A. (2021), The impact of collective bargaining on employment and wage inequality: evidence from a new taxonomy of bargaining systems, *European Journal of Industrial Relations 27*, 185–202.

Gerber, A. and Upmann, T. (2006), Bargaining solutions at work: qualitative differences in policy implications, *Mathematical Social Sciences 52*, 162–175.

Gertler, M. and Trigari, A. (2009), Unemployment fluctuations with staggered Nash wage bargaining, *Journal of Political Economy 117*, 38–86.

Goerke, L. (1996), Taxes on payroll, revenues and profits in three models of collective bargaining, *Scottish Journal of Political Economy 43*, 549–565.

Grandner, T. (2001), Unions in oligopolistic, vertically connected industries, *European Economic Review 45*, 1723–1740.

Hayter, S., Fashoyin, T., and Kochan, T. A. (2011), Review essay: collective bargaining for the 21st century, *Journal of Industrial Relations 53*, 225–247.

Horn, H. and Svensson, L. E. O. (1986), Trade unions and optimal labour contracts, *Economic Journal 96*, 323–341.

International Labour Office (2015), Collective bargaining in the public service in the European Union, ILO Working Paper 309, International Labour Office, Geneva.

Jacquet, L., Lehmann, E., and Van der Linden, B. (2014), Optimal income taxation with Kalai wage bargaining and endogenous participation, *Social Choice and Welfare 42*, 381–402.

Jäger, S., Schoefer, B., Young, S., and Zweimüller, J. (2020), Wages and the value of nonemployment, *Quarterly Journal of Economics 135*, 1905–1963.

Kalai, E. (1977), Proportional solutions to bargaining situations: interpersonal utility comparisons, *Econometrica 45*, 1623–1630.

Kalai, E. and Smorodinsky, M. (1975), Other solutions to Nash's bargaining problem, *Econometrica 43*, 513–518.

Kraft, K. (2006), Wage versus efficient bargaining in oligopoly, *Managerial and Decision Economics 27*, 595–604.

Krusell, P., Mukoyama, T., and Sahin, A. (2010), Labour-market matching with precautionary savings and aggregate fluctuations, *Review of Economic Studies 77*, 1477–1507.

Lawson, N. P. (2011), Is collective bargaining Pareto efficient? A survey of the literature, *Journal of Labor Research 32*, 282–304.

l'Haridon, O., Malherbet, F., and Perez-Duarte, S. (2013), Does bargaining matter in the small firms matching model?, *Labour Economics 21*, 42–58.

Liu, D., Lv, W., Li, H., and Tang, J. (2017), Bargaining model of labor disputes considering social mediation and bounded rationality, *European Journal of Operational Research 262*, 1064–1071.
McDonald, I. M. and Solow, R. M. (1981), Wage bargaining and employment, *American Economic Review 71* (5), 896–908.
Müller, J. and Upmann, T. (2018), Centralised labour market negotiations: strategic behaviour curbs employment, *Journal of Institutional and Theoretical Economics 174*, 278–302.
Nash, J. F. (1950), The bargaining problem, *Econometrica 18*, 155–162.
OECD (2019a), *Negotiating Our Way Up: Collective Bargaining in a Changing World of Work*, OECD Publishing, Paris.
OECD (2019b), *OECD Employment Outlook 2019: The Future of Work*, OECD Publishing, Paris.
Oswald, A. J. (1985), The economic theory of trade unions: an introductory survey, *Scandinavian Journal of Economics 87*, 160–193.
Paz Espinosa, M. and Rhee, C. (1989), Efficient wage bargaining as a repeated game, *Quarterly Journal of Economics 104*, 565–588.
Petrakis, E. and Vlassis, M. (2000), Endogenous scope of bargaining in a union-oligopoly model: when will firms and unions bargain over employment?, *Labour Economics 7*, 261–281.
Ranjan, P. (2013), Offshoring, Unemployment, and wages: the role of labor market institutions, *Journal of International Economics 89*, 172–186.
Santoni, M. (2014), Product market integration and wage bargaining institutions, *Labour Economics 27*, 1–15.
Schulten, T. (2020), Collective bargaining in Germany 2020: annual report of the WSI collective agreement archive, WSI Collective Agreement Archive, https://www.boeckler.de/pdf/p_ta_jb_2020_english.pdf.
Strand, J. (2002), Wage bargaining and turnover costs with heterogenous labor and perfect history screening, *European Economic Review 46*, 1209–1227.
Svejnar, J. (1986), Bargaining power, fear of disagreement, and wage settlements: theory and evidence from U.S. industry, *Econometrica 54*, 1055–1078.
Thomson, W. (1981), Nash's bargaining solution and utilitarian choice rules, *Econometrica 49*, 535–538.
Upmann, T. (2009), A positive analysis of labor-market institutions and tax reforms, *International Tax and Public Finance 16*, 621–646.
Upmann, T. and Müller, J. (2014), The structure of firm-specific labour unions, *Journal of Institutional and Theoretical Economics 170*, 336–364.
Walsh, F. (2012), Efficiency wages and bargaining, *Oxford Economic Papers 64*, 635–654.
Wehke, S. (2009), Union wages, hours of work and the effectiveness of partial coordination agreements, *Labour Economics 16*, 89–96.
Zhao, L. (1995), Cross-hauling direct foreign investment and unionized oligopoly, *European Economic Review 39*, 1237–1253.

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