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Do we need non-Boussinesq effects in an ocean general circulation model for climate simulations?

Martin Losch (Alfred-Wegener-Institut, Bremerhaven)
Jean-Michel Campin (MIT, Cambridge, MA)

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This is an over 20 year old discussion:

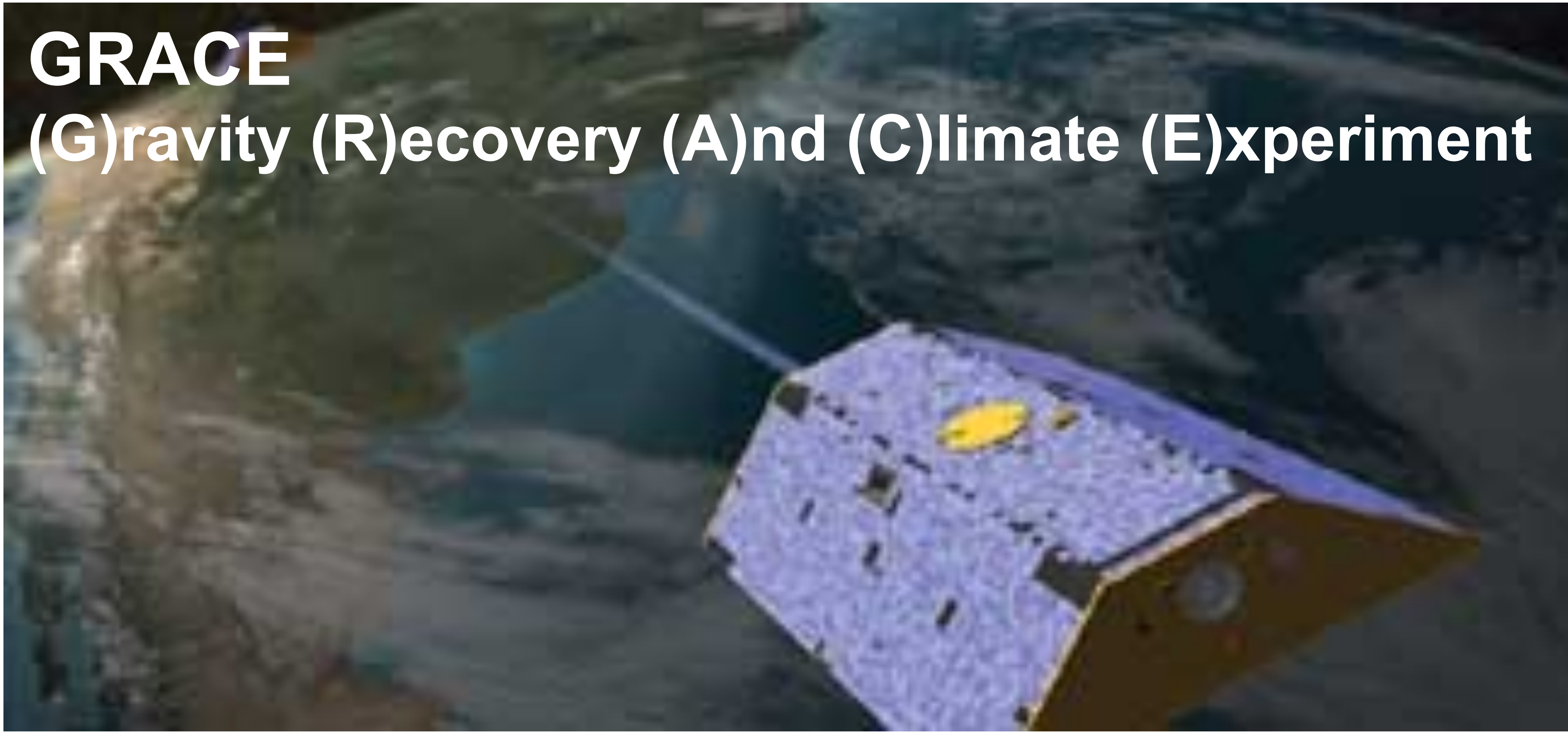
Lu (2001)(12); McDougall, **Greatbatch**, Lu (2002)(30),

Greatbatch, Lu, Cai (2001)(30);

Huang et al. (2001)(38);

de Szoeke and Samelson (2002)(36), Losch, Adcroft, Campin (2004)(34)

GRACE
(G)ravity (R)ecovery (A)nd (C)limate (E)xperiment



- Very accurately measures gravity
- Can infer changes in mass distribution in oceans
- Boussinesq models conserve volume not mass
- How can we test whether the difference matters?

Boussinesq Approximation

According to Spiegel and Veronis (1960):

1. The fluctuations in density which appear with the advent of motion result principally from thermal (as opposed to pressure) effects.
2. In the equations for the rate of change of momentum and mass, density variations may be neglected except when they are coupled to the gravitational acceleration in the buoyancy force.

$$1. \quad \frac{1}{\rho}\frac{D\rho}{Dt} + \nabla \cdot \mathbf{v} = 0 \Rightarrow \nabla \cdot \mathbf{v} = 0$$

mass balance becomes volume balance

$$2. \quad \rho_0 \frac{D\mathbf{v}}{Dt} + \rho_0 f(\mathbf{k} \times \mathbf{v}) = -\nabla p - \rho g \mathbf{k} + \rho_0 \mathcal{F}$$

One consequence of the Boussinesq Approximation

$$\int_{-H}^{\eta} \left(\frac{1}{\rho} \frac{D\rho}{Dt} + \nabla \cdot \mathbf{u} \right) dz = \frac{Q_{FW}}{\rho_c}$$

$$\Rightarrow \frac{\partial \overline{\eta}}{\partial t} = \overline{\frac{Q_{FW}}{\rho_c}} - \overline{\int_{-H}^{\eta} \frac{1}{\rho} \frac{D\rho}{Dt} \, dz}$$

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=> any sea level study should use non-Boussinesq models (but the global mean can be recovered accurately a-posteriori: Greatbatch, 1994).

How to include non-Boussinesq effects?

various methods for integrating the full continuity equation

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = 0 \Leftrightarrow \nabla \cdot \mathbf{u} = -\frac{1}{\rho}\frac{D\rho}{Dt}$$

- modify/reinterpret existing codes:
  - Lu (2001); McDougall, Greatbatch, Lu (2002), implemented in Greatbatch, Lu, Cai (2001);
  - de Szoeke and Samelson (2002): exploit duality between Boussinesq and non-Boussinesq equations, implemented in MITgcm (Losch, Adcroft, Campin, 2004)
- write new non-Boussinesq models (from scratch)
  - in pressure coordinates: Huang et al. (2001);
  - σ-model: Song et al. (2004, 2006, 2010, …)
  - new non-Boussinesq algorithm: Auclair et al (2018)

more non-Boussinesq effects

- various estimates of size of effects, generally larger than previously assumed
  - According to McDougall, Greatbatch, Lu (2002): “On Conservation Equations in Oceanography: How Accurate Are Boussinesq Ocean Models?” Davies (1994): “Diapycnal mixing in the ocean: Equations for large-scale budgets”, errors of order of diagpycnal mixing occur in Reynolds averaged equations when replacing density by a constant:

$$\partial_t(\rho C) + \nabla \cdot (\rho \mathbf{u} C) = \nabla \cdot (\rho \kappa_C \nabla C)$$

with $\partial_t C + \nabla \cdot (\mathbf{u} C) \approx \nabla \cdot (\kappa_C \nabla C)$

RA: $\partial_t \overline{C} + \overline{\mathbf{u}} \cdot \nabla \overline{C} \approx - \nabla \cdot (\overline{\mathbf{u}' C'})$

non-Boussinesq equations: Z-coordinate approach (McDougall et al 2002, implemented in Greatbatch et al 2001)

Problem for Reynolds averaged equations: $\overline{C}_t + \overline{u} \cdot \nabla \overline{C} = -\nabla \cdot \left(\overline{u'C'}\right)$

Solution: interpret variables as density weighted means

$$\overline{\mathbf{u}}^\rho = \overline{\rho \mathbf{u}}/\overline{\rho}, \quad \tilde{\overline{\mathbf{u}}} = \overline{\rho}\,\overline{\mathbf{u}}^\rho/\rho_0 = \overline{\rho \mathbf{u}}/\rho_0, \quad \overline{C}^\rho = \overline{\rho C}/\overline{\rho}$$

$$\left(\frac{\overline{\rho}}{\rho_o}\right)_t + \nabla \cdot \tilde{\overline{\mathbf{u}}} = 0,$$

$$\left(\frac{\overline{\rho}}{\rho_o}\overline{C}^\rho\right)_t + \nabla \cdot \left(\tilde{\overline{\mathbf{u}}}\,\overline{C}^\rho\right) = \nabla \cdot \left(\mathbf{K}\nabla\overline{C}^\rho\right),$$

$$\tilde{\overline{\mathbf{u}}}_t + \nabla \cdot \left(\frac{\rho_o}{\overline{\rho}}\tilde{\overline{\mathbf{u}}}\,\tilde{\overline{\mathbf{u}}}\right) + 2\mathbf{\Omega} \times \tilde{\overline{\mathbf{u}}} = -\frac{1}{\rho_o}\nabla\overline{p} - \mathbf{k}g\frac{\overline{\rho}}{\rho_o} + \nabla \cdot \left(\mathbf{A}\nabla\frac{\rho_o}{\overline{\rho}}\tilde{\overline{\mathbf{u}}}\right).$$

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non-Boussinesq pressure coordinates

$$\frac{D\rho}{Dt} + \rho\left(\nabla_z \cdot \mathbf{u} + \frac{\partial w}{\partial z}\right) = 0 \qquad (2.9)$$

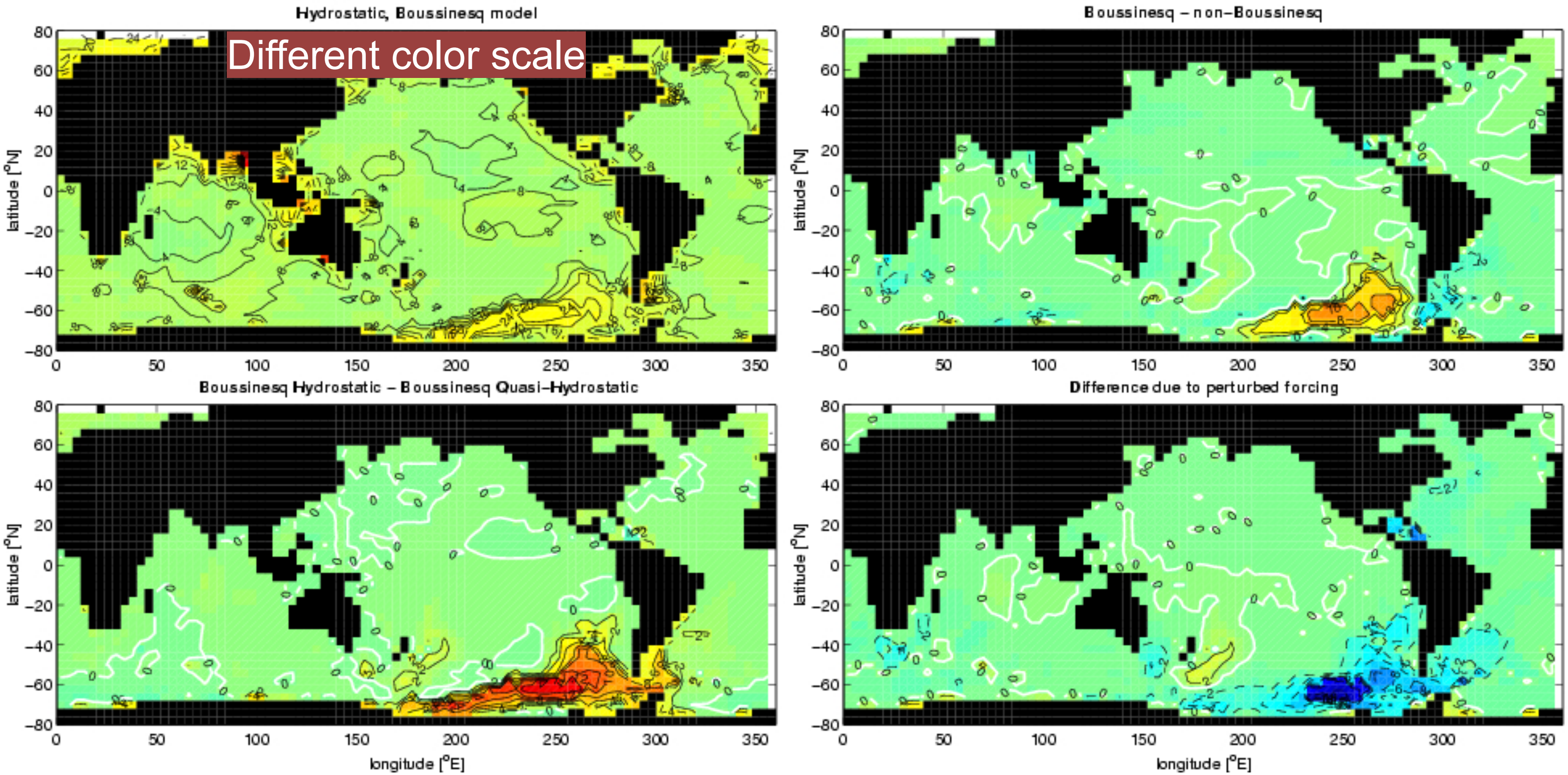
is transformed to a general coordinate p (not necessarily pressure) that replaces z, it becomes (appendix A)

$$\frac{D}{Dt}(\rho z_p) + \rho z_p\left(\nabla_p \cdot \mathbf{u} + \frac{\partial \omega}{\partial p}\right) = 0 \qquad (2.10)$$

with hydrostatic pressure $\rho z_p = \rho\frac{\partial z}{\partial p} = -\frac{1}{g} =$ constant

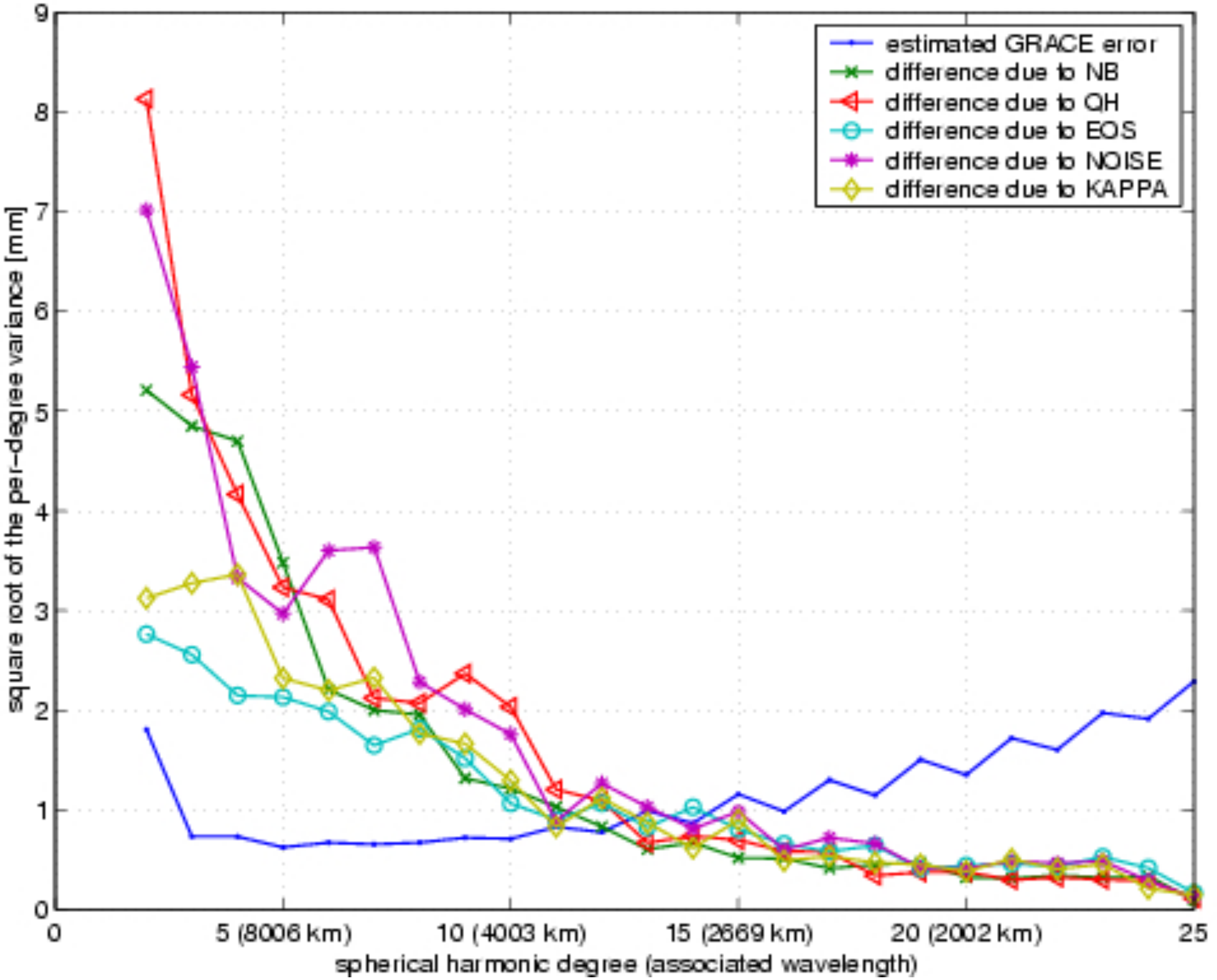
de Szoeke and Samelson (2002)

Variability in Bottom Pressure (cm)



- 4˚ model, between 80˚N/S, 15 levels, no sea ice, simple convective adjustment, no eddy parameterisation scheme, nonlinear free surface (Losch et al, 2004)

How important are these effects?



Losch et al. (2004), 4deg grid

- < 2000km
  - error in GRACE data larger than effects
- > 2000km
  - effects in model no bigger than due to numerical noise!
- coarse resolution!!!!
- *Story will change at higher resolution*

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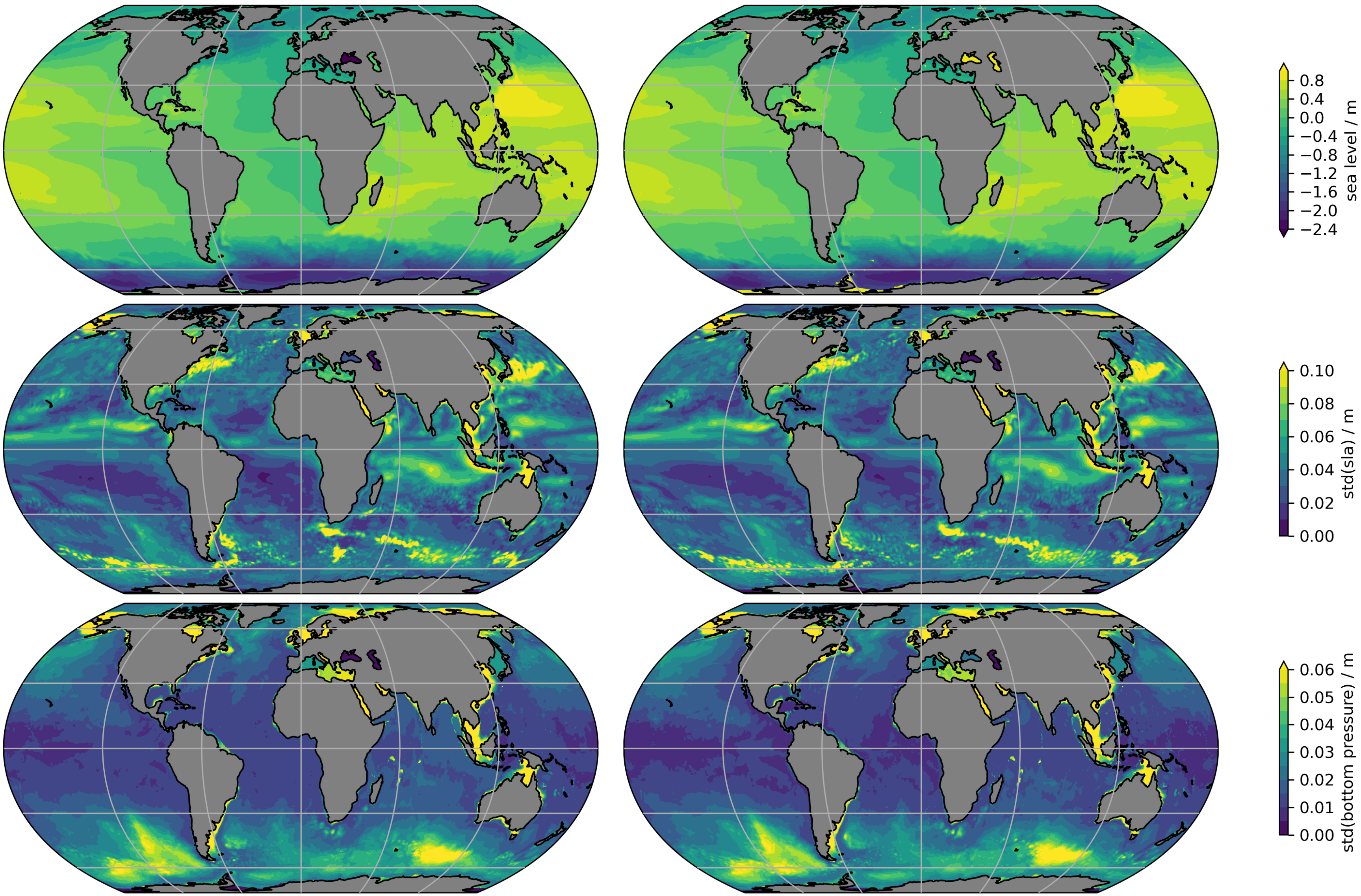
for a coarse resolution general circulation model

- Boussinesq and hydrostatic approximations have similar effects on the circulation
- effects due to numerical truncation error and unclear parameterisations are of similar magnitude
- even coarse models are sensitive to small changes in dynamics and forcing!!!!!

Update: increase complexity and resolution

- LLC270 (1/3 °), 50 levels, some “eddies”
- sea ice model (levitating)
- Gent-McWilliams and Redi-scheme ($\kappa_{GM} = 80\,\mathrm{ms}^{-2}$)
- Vertical mixing scheme: TKE (Gaspar et al, 1990) + IDEMIX (Olbers and Eden, 2013, Eden and Olbers, 2014)
- (new TEOS10 equation of state, unfinished)

z vs p-coordinates (year 62)



z-coordinates (Boussinesq) p-coordinates (non-Boussinesq)

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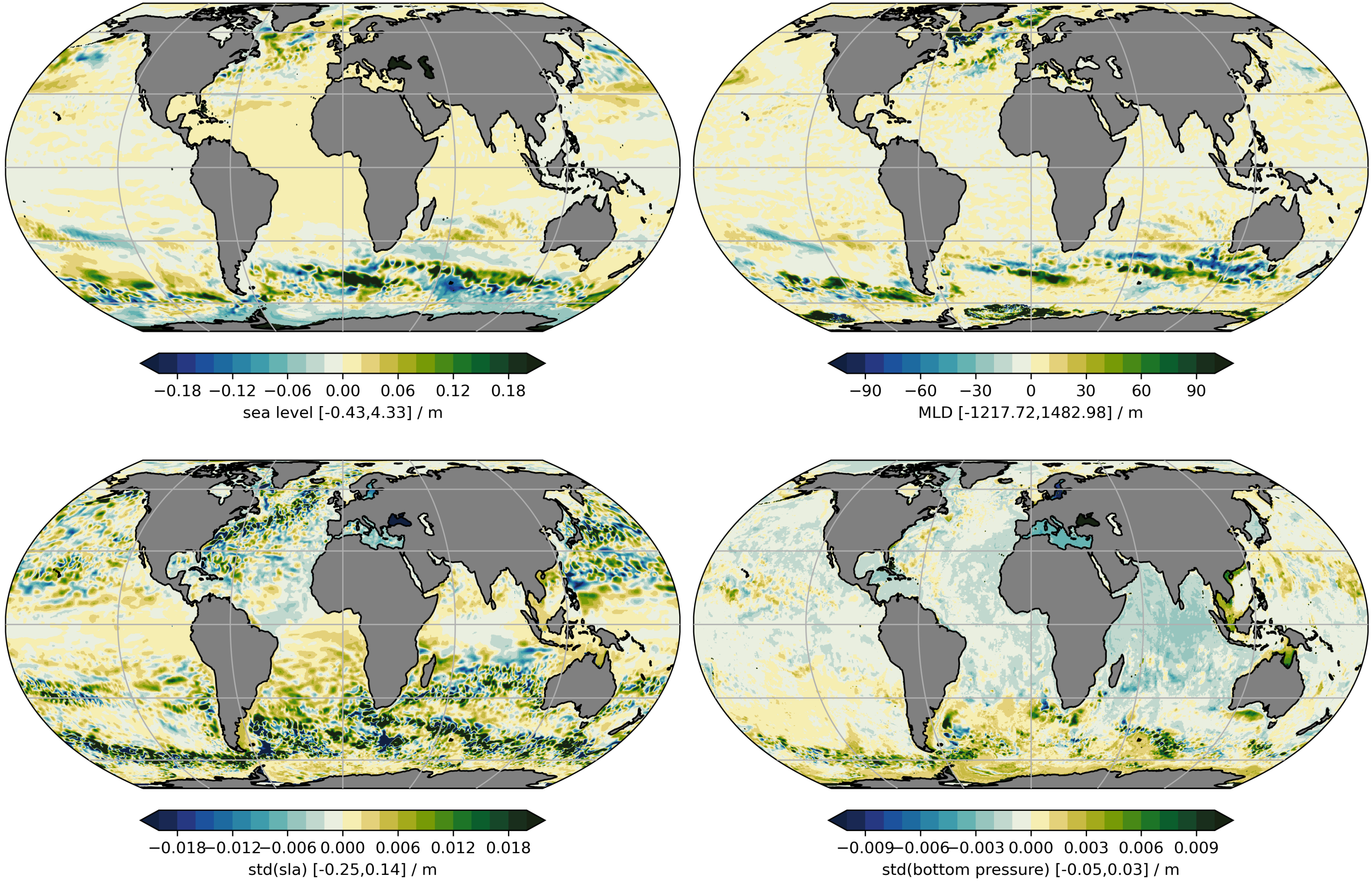
differences small but systematic?



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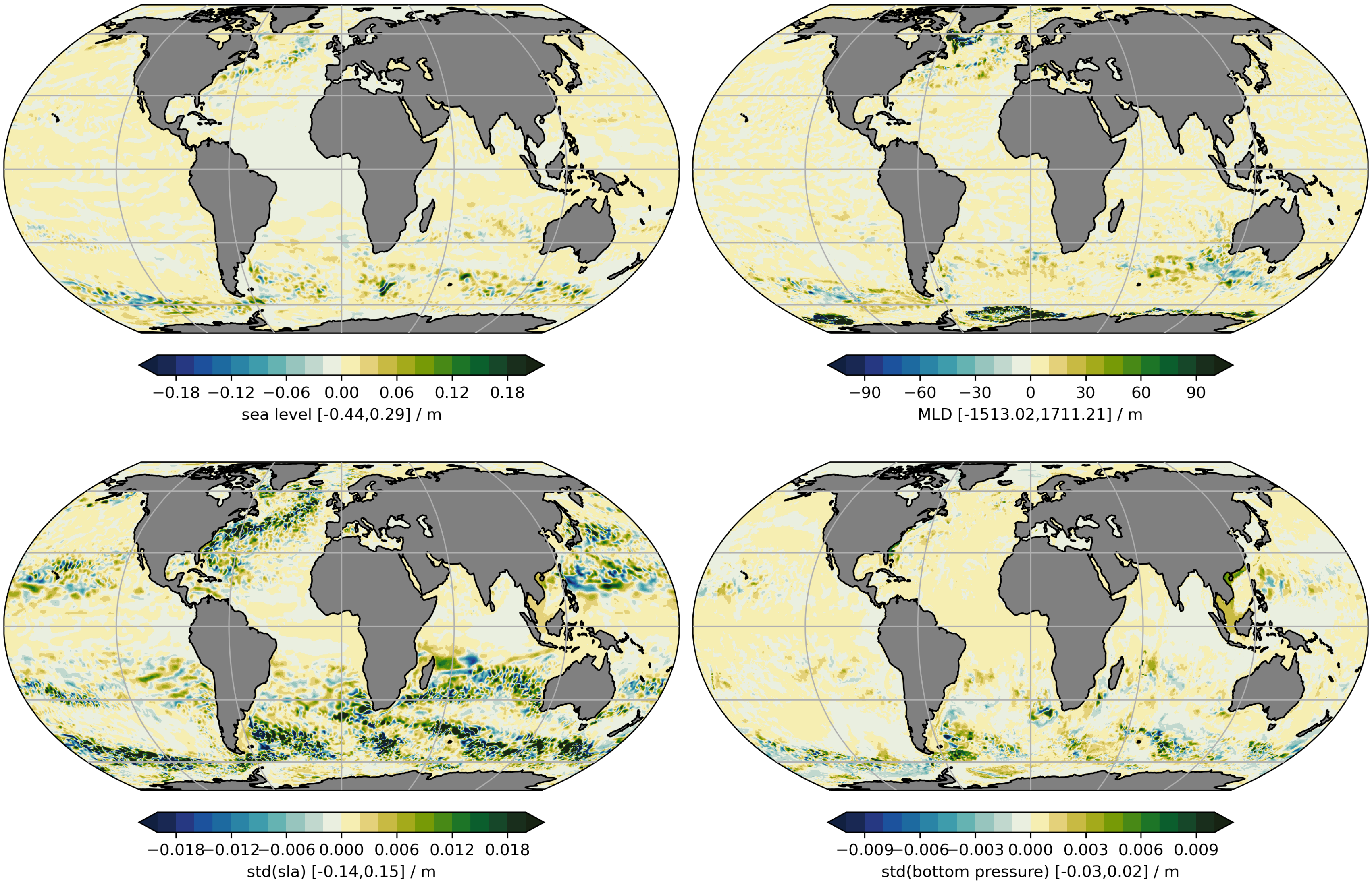
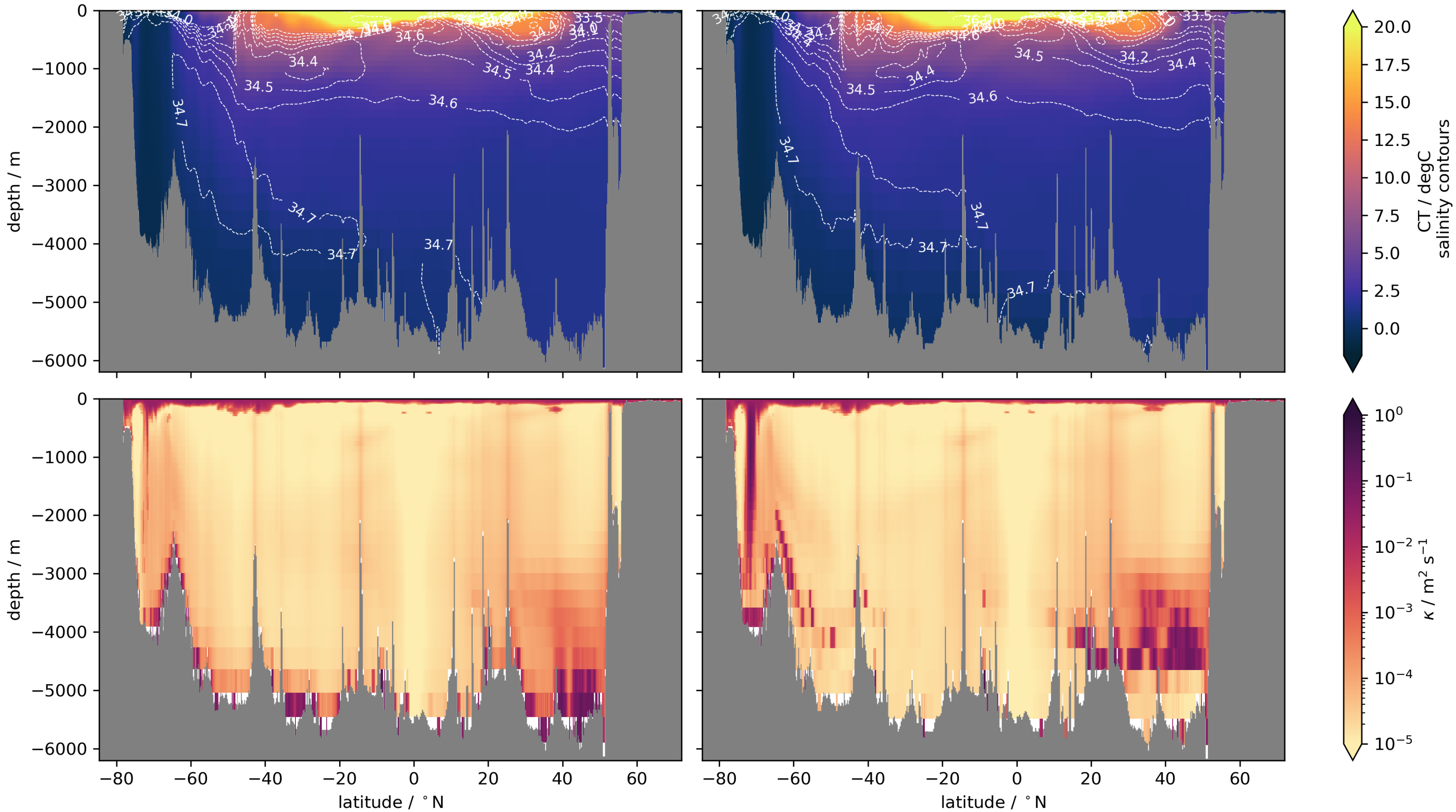
Difference due to model numerics



z vs p-coordinates (year 62)

- with IDEMIX!!!
- section through the Pacific Ocean



z-coordinates (Boussinesq) p-coordinates (non-Boussinesq)

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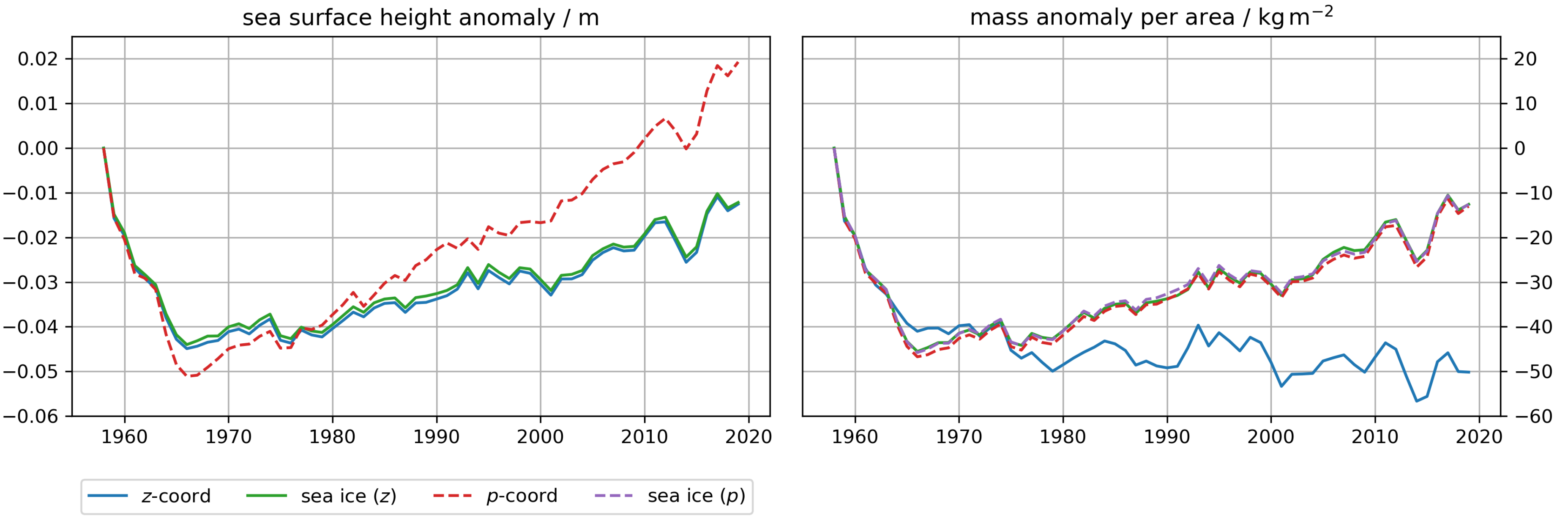
Mean volume and mass



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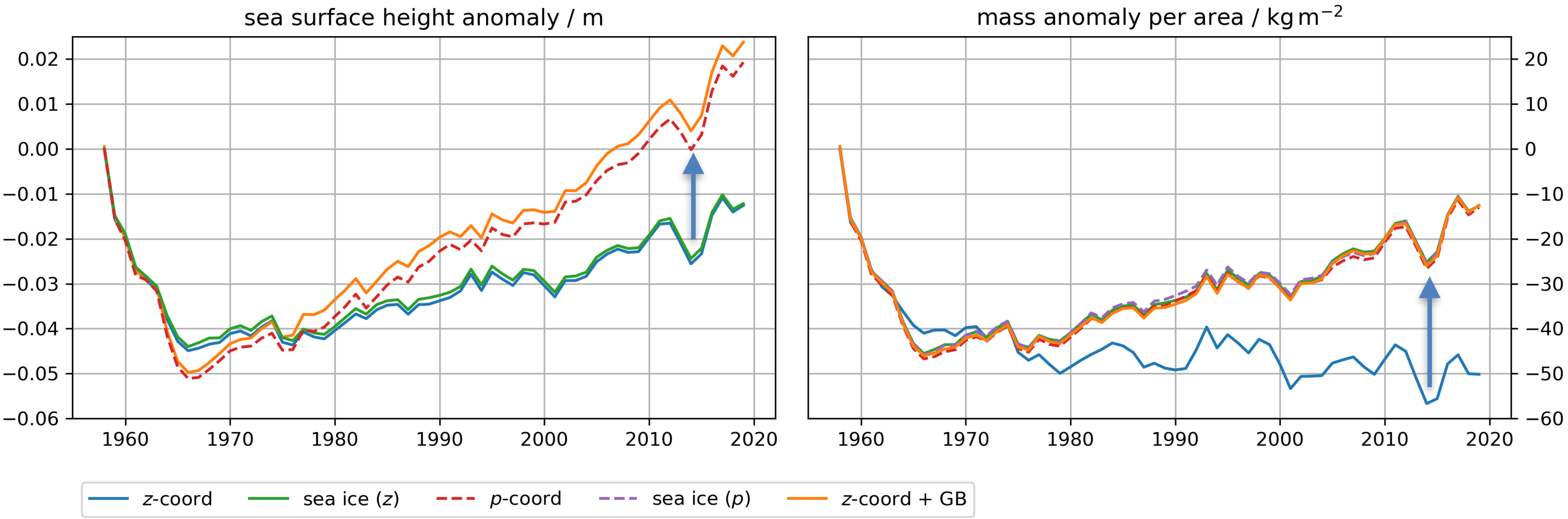
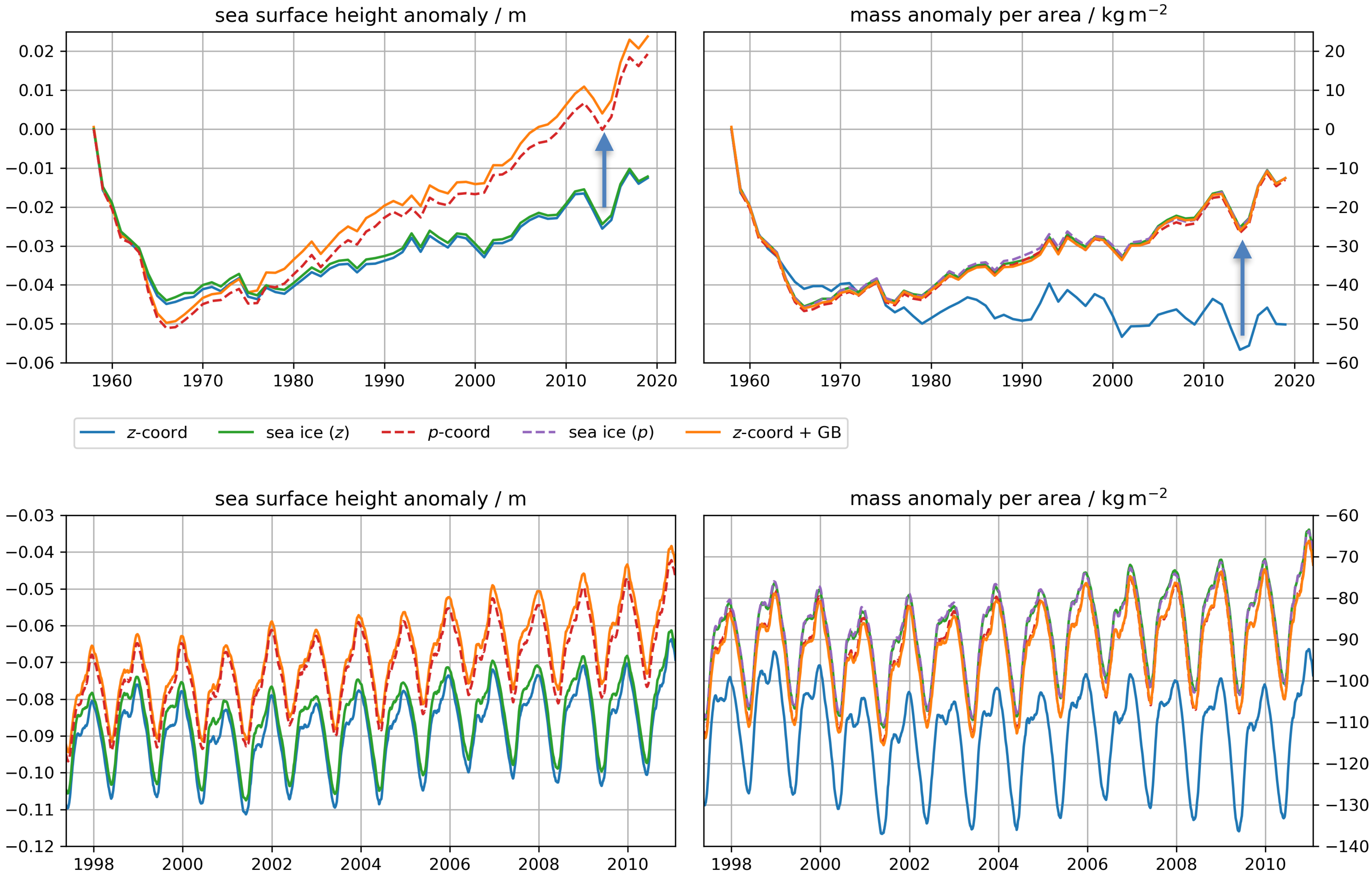
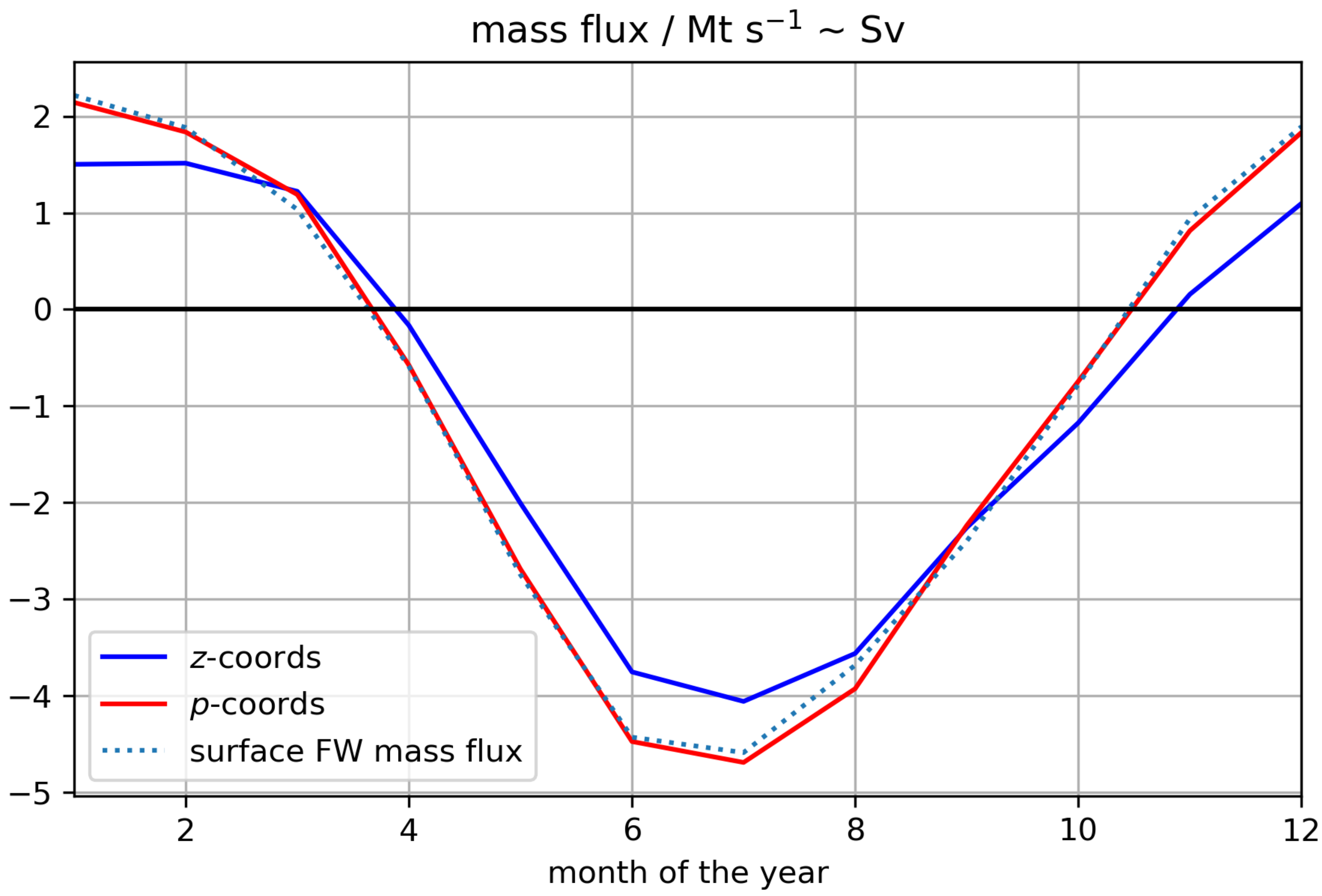


Mean volume and mass



Seasonal equatorial transport



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at higher resolution

- Greatbatch (1994) correction is still great
- differences between non-Boussinesq and Boussinesq model may be larger at higher resolution, maybe even systematic, but still at the level of other uncertainties (here, EOS) (But more careful comparison required: initial conditions, quasi-hydrostatic approximation)
- Order (10%) of cross-equator mass transport not resolved in Boussinesq model
- Replacing pressure by mass coordinates (gp) conveniently solves forcing issue by atmospheric pressure (to be done)