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Do we need non-Boussinesq effects in an ocean general circulation model for climate simulations?

Martin Losch (Alfred-Wegener-Institut, Bremerhaven)
Jean-Michel Campin (MIT, Cambridge, MA)

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This is an over 20 year old discussion:

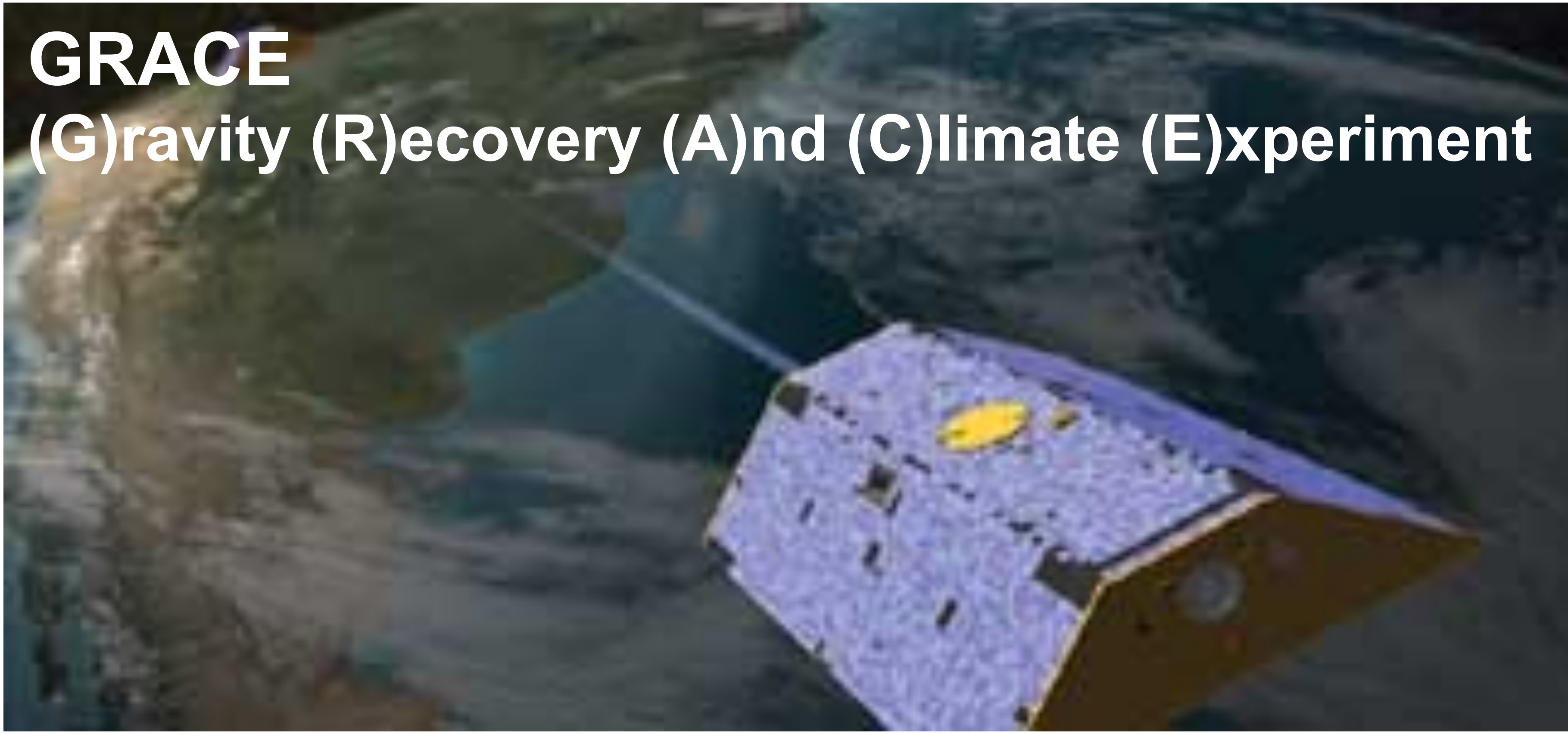
Lu (2001)(12); McDougall, Greatbatch, Lu (2002)(30),

Greatbatch, Lu, Cai (2001)(30);

Huang et al. (2001)(38);

de Szoeke and Samelson (2002)(36), Losch, Adcroft, Campin (2004)(34)

GRACE
(G)ravity (R)ecovery (A)nd (C)limate (E)xperiment



- Very accurately measures gravity
- Can infer changes in mass distribution in oceans
- Boussinesq models conserve volume not mass
- How can we test whether the difference matters?

Review: hydrostatic approximation

- aspect ratio = (vertical scale)/(horizonal scale) is small
- vertical acceleration due to gravity is balanced by vertical pressure gradient

$$\frac{Du}{Dt} = -\underline{\frac{uw}{R}} + \frac{uv\tan\phi}{R} \quad \underline{-2\Omega w\cos\phi} \quad +2\Omega v\sin\phi \quad -\frac{\partial}{\partial x}\frac{p}{\rho_0} + F_u$$

$$\frac{Dv}{Dt} = -\underline{\frac{vw}{R}} + \frac{u^2\tan\phi}{R} \quad -2\Omega u\sin\phi \quad -\frac{\partial}{\partial y}\frac{p}{\rho_0} + F_v$$

$$\underline{\underline{\frac{Dw}{Dt}}} = -\underline{\frac{u^2+v^2}{R}} \quad \underline{-2\Omega u\cos\phi} \quad -\frac{1}{\rho_0}\left(g\rho + \frac{\partial p}{\partial z}\right) + \underline{\underline{F_w}}$$

metric terms Coriolis terms

Marshall et al. (1997): hydrostatic, quasi-hydrostatic, non-hydrostatic

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$$\frac{Du}{Dt} = \frac{uv\tan\phi}{R} + 2\Omega\, v\sin\phi - \frac{\partial}{\partial x}\frac{p}{\rho_0} + F_u$$

$$\frac{Dv}{Dt} = \frac{u^2\tan\phi}{R} - 2\Omega\, u\sin\phi - \frac{\partial}{\partial y}\frac{p}{\rho_0} + F_v$$

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metric terms

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Marshall et al. (1997): hydrostatic, quasi-hydrostatic, non-hydrostatic

Boussinesq Approximation

According to Spiegel and Veronis (1960):

1. The fluctuations in density which appear with the advent of motion result principally from thermal (as opposed to pressure) effects.
2. In the equations for the rate of change of momentum and mass, density variations may be neglected except when they are coupled to the gravitational acceleration in the buoyancy force.

$$1. \quad \frac{1}{\rho}\frac{D\rho}{Dt} + \nabla \cdot \mathbf{v} = 0 \Rightarrow \nabla \cdot \mathbf{v} = 0$$

mass balance becomes volume balance

$$2. \quad \rho_0 \frac{D\mathbf{v}}{Dt} + \rho_0 f(\mathbf{k} \times \mathbf{v}) = -\nabla p - \rho g \mathbf{k} + \rho_0 \mathcal{F}$$

One consequence of the Boussinesq Approximation

$$\int_{-H}^{\eta} \left(\frac{1}{\rho} \frac{D\rho}{Dt} + \nabla \cdot \mathbf{u} \right) dz = \frac{Q_{FW}}{\rho_c}$$

$$\Rightarrow \frac{\partial \overline{\eta}}{\partial t} = \overline{\frac{Q_{FW}}{\rho_c}} - \overline{\int_{-H}^{\eta} \frac{1}{\rho} \frac{D\rho}{Dt} dz}$$

One consequence of the Boussinesq Approximation

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$$\Rightarrow \frac{\partial\overline{\eta}}{\partial t}=\overline{\frac{Q_{FW}}{\rho_c}}-\overline{\int_{-H}^{\eta}\frac{1}{\rho}\frac{D\rho}{Dt}dz}$$

=> any sea level study should use non-Boussinesq models (but the global mean can be recovered accurately a-posteriori: Greatbatch, 1994).

Additive correction to mean sea leavel: $h_0\left(1-\frac{\overline{\rho}(t)}{\rho_0}\right)$

More Consequences of the Boussinesq Approximation

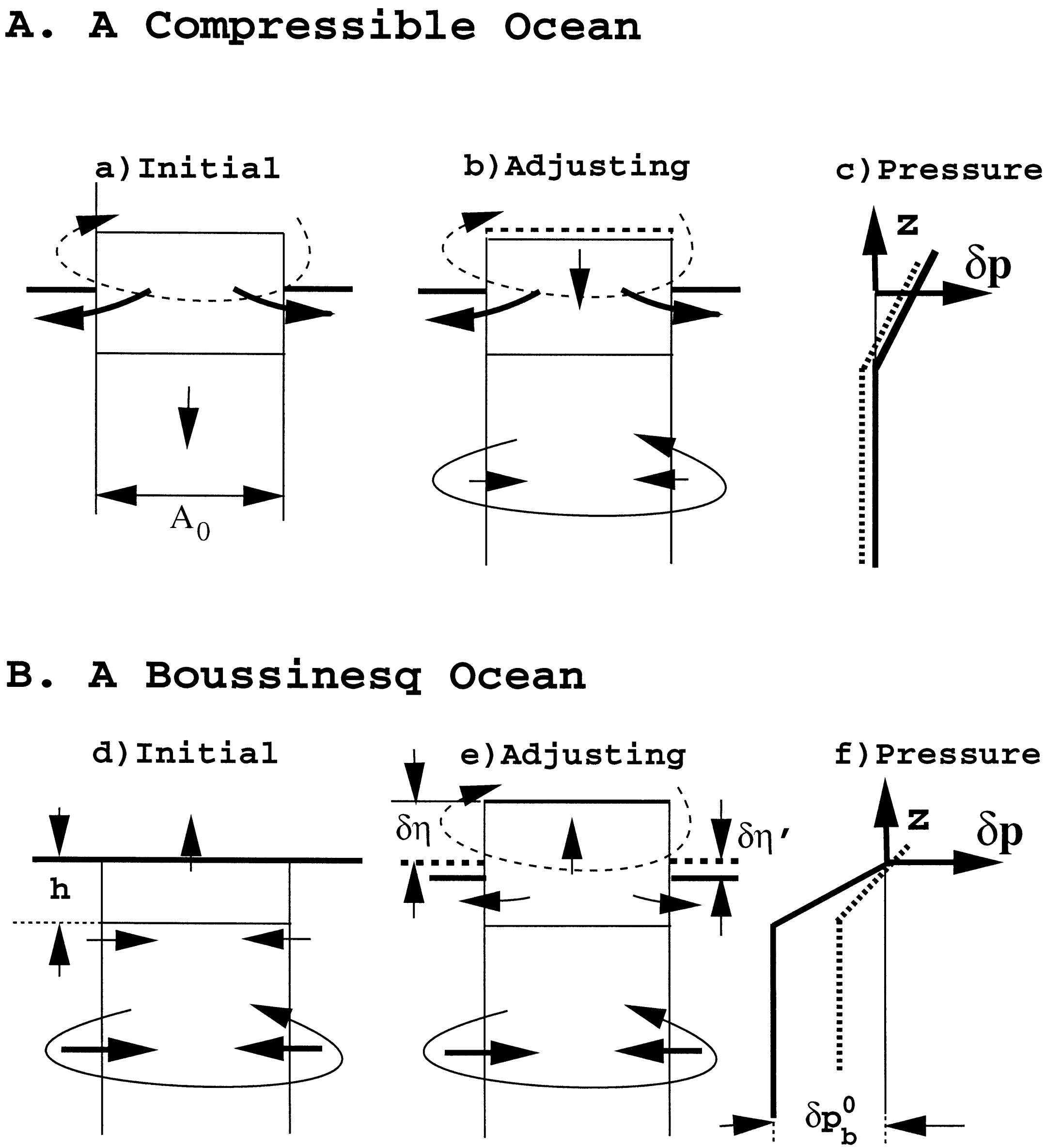

FIG. 1. Sketch of geostrophic adjustment in response to surface heating.

example of Boussinesq effect: geostrophic adjustment after surface heating,
Huang and Jin (2002)

eventually leads to Goldbrough/Stommel gyres; forcing is an order of magnitude smaller than for direct E-P forcing

How to include non-Boussinesq effects?

various methods for integrating the full continuity equation

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = 0 \Leftrightarrow \nabla \cdot \mathbf{u} = -\frac{1}{\rho}\frac{D\rho}{Dt}$$

- modify/reinterpret existing codes:
  - Lu (2001); McDougall, Greatbatch, Lu (2002), implemented in Greatbatch, Lu, Cai (2001);
  - de Szoeke and Samelson (2002): exploit duality between Boussinesq and non-Boussinesq equations, implemented in MITgcm (Losch, Adcroft, Campin, 2004)
- write new non-Boussinesq models (from scratch)
  - in pressure coordinates: Huang et al. (2001);
  - σ-model: Song et al. (2004, 2006, 2010, …)
  - new non-Boussinesq algorithm: Auclair et al (2018)

more non-Boussinesq effects

- various estimates of size of effects, generally larger than previously assumed
  - According to McDougall, Greatbatch, Lu (2002): “On Conservation Equations in Oceanography: How Accurate Are Boussinesq Ocean Models?” Davies (1994): “Diapycnal mixing in the ocean: Equations for large-scale budgets”, errors of order of diagpycnal mixing occur in Reynolds averaged equations when replacing density by a constant:

$$\partial_t(\rho C) + \nabla \cdot (\rho \mathbf{u} C) = \nabla \cdot (\rho \kappa_C \nabla C)$$

with $\partial_t C + \nabla \cdot (\mathbf{u} C) \approx \nabla \cdot (\kappa_C \nabla C)$

RA: $\partial_t \overline{C} + \overline{\mathbf{u}} \cdot \nabla \overline{C} \approx - \nabla \cdot (\overline{\mathbf{u}' C'})$

non-Boussinesq equations: Z-coordinate approach (McDougall et al 2002, implemented in Greatbatch et al 2001)

Problem for Reynolds averaged equations: $\overline{C}_t + \overline{u} \cdot \nabla \overline{C} = -\nabla \cdot \left(\overline{u'C'}\right)$

Solution: interpret variables as density weighted means

$$\overline{\mathbf{u}}^{\rho} = \overline{\rho \mathbf{u}}/\overline{\rho}, \quad \tilde{\overline{\mathbf{u}}} = \overline{\rho}\,\overline{\mathbf{u}}^{\rho}/\rho_0 = \overline{\rho \mathbf{u}}/\rho_0, \quad \overline{C}^{\rho} = \overline{\rho C}/\overline{\rho}$$

$$\left(\frac{\overline{\rho}}{\rho_o}\right)_t + \boldsymbol{\nabla} \cdot \tilde{\overline{\mathbf{u}}} = 0,$$

$$\left(\frac{\overline{\rho}}{\rho_o}\overline{C}^{\rho}\right)_t + \boldsymbol{\nabla} \cdot (\tilde{\overline{\mathbf{u}}}\,\overline{C}^{\rho}) = \boldsymbol{\nabla} \cdot (\mathbf{K}\boldsymbol{\nabla}\overline{C}^{\rho}),$$

$$\tilde{\overline{\mathbf{u}}}_t + \boldsymbol{\nabla} \cdot \left(\frac{\rho_o}{\overline{\rho}}\tilde{\overline{\mathbf{u}}}\,\tilde{\overline{\mathbf{u}}}\right) + 2\boldsymbol{\Omega} \times \tilde{\overline{\mathbf{u}}} = -\frac{1}{\rho_o}\boldsymbol{\nabla}\overline{p} - \mathbf{k}g\frac{\overline{\rho}}{\rho_o} + \boldsymbol{\nabla} \cdot \left(\mathbf{A}\boldsymbol{\nabla}\frac{\rho_o}{\overline{\rho}}\tilde{\overline{\mathbf{u}}}\right).$$

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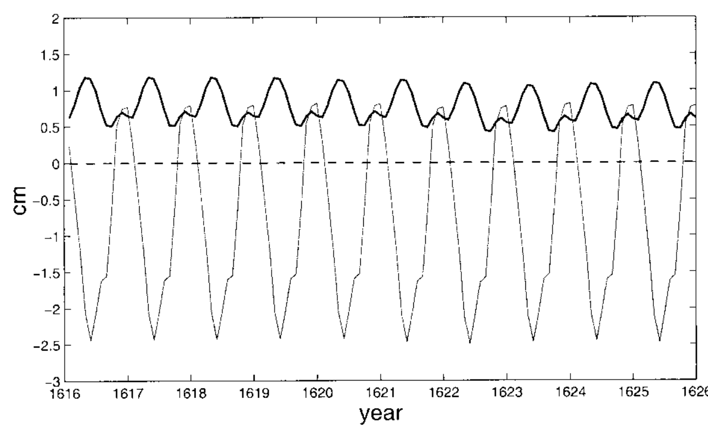

FIG. 4. Time evolution of the domain-averaged SSH from the coarse-resolution global ocean simulation. The dashed line depicts the solution of the Boussinesq run; the thicker solid line depicts the non-Boussinesq run with the "virtual" salt flux surface boundary condition; and the thinner solid line depicts the non-Boussinesq run with the "real freshwater flux" surface boundary condition.

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non-Boussinesq pressure coordinates

$$\frac{D\rho}{Dt} + \rho\left(\nabla_z \cdot \mathbf{u} + \frac{\partial w}{\partial z}\right) = 0 \qquad (2.9)$$

is transformed to a general coordinate p (not necessarily pressure) that replaces z, it becomes (appendix A)

$$\frac{D}{Dt}(\rho z_p) + \rho z_p\left(\nabla_p \cdot \mathbf{u} + \frac{\partial \omega}{\partial p}\right) = 0 \qquad (2.10)$$

with hydrostatic pressure $\rho z_p = \rho \frac{\partial z}{\partial p} = -\frac{1}{g} = \text{constant}$

de Szoeke and Samelson (2002)

non-Boussinesq eq: The z-p isomorphism

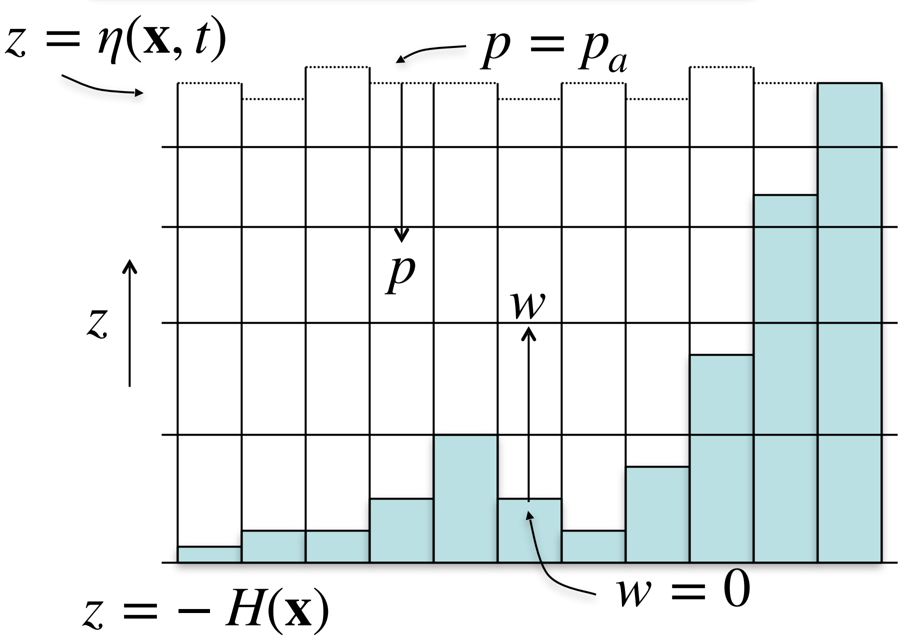
Ocean (z coordinates)

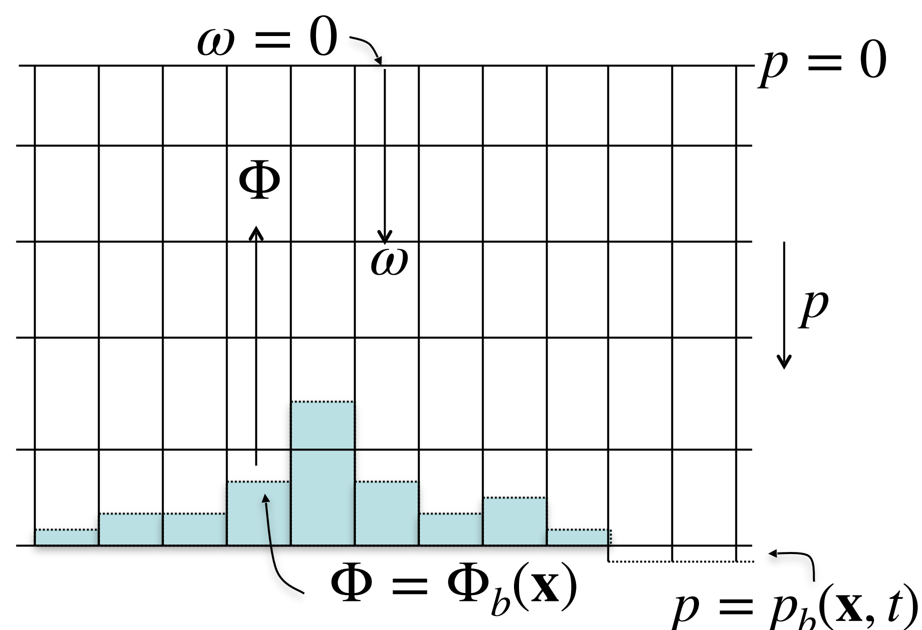
$$D_t\mathbf{u} + 2\Omega\times\mathbf{u} + \frac{1}{\rho}\nabla_z p = \mathbf{F}$$
$$g\rho + \partial_z p = 0$$
$$\nabla_z \mathbf{u} + \partial_z w = 0$$
$$\partial_t \eta + \nabla\cdot(H+\eta)\langle\mathbf{u}\rangle = P - E$$
$$D_t\theta = Q_\theta$$
$$D_t s = Q_s$$
$$\rho = \rho(s,\theta,p)$$

$$z \leftrightarrow p$$
$$p/\rho_0 \leftrightarrow \Phi$$
$$g\rho \leftrightarrow \alpha$$
$$w \leftrightarrow \omega$$
$$\eta + H \leftrightarrow p_b$$

$$D_t\mathbf{u} + 2\Omega\times\mathbf{u} + \nabla_p\Phi = \mathbf{F}$$
$$\alpha + \partial_p\Phi = 0$$
$$\nabla_p\mathbf{u} + \partial_p\omega = 0$$
$$\partial_t p_b + \nabla\cdot p_b\langle\mathbf{u}\rangle = g\rho(P-E)$$
$$D_t\theta = Q_\theta$$
$$D_t s = Q_s$$
$$\alpha = \alpha(s,\theta,p)$$

Atmosphere/Ocean (in p)





(deSoeke and Samelson, 2002, Marshall et al. 2003)

This is how MITgcm does it:

- generalised r-coordinates

$$D_t\mathbf{u} + 2\Omega \times \mathbf{u} + \nabla_r \phi = \mathbf{F} \quad \text{horizontal mom.}$$

$$b + \partial_r \phi = 0 \quad \text{hydrostatic eq.}$$

$$\nabla_r \mathbf{u} + \partial_r \dot{r} = 0 \quad \text{continuity eq.}$$

$$\partial_t r_s + \nabla_r \cdot \int_{-R_{\text{fixed}}}^{r_s} \mathbf{u}\, dr = \gamma\,(P - E) \quad \text{free surface}$$

$$D_t\theta = Q_\theta \quad \text{tracer equations}$$

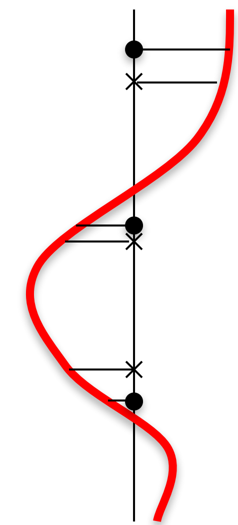
$$D_t s = Q_s$$

$$b = EOS$$

$r = z,\, p$	$b = -g\dfrac{\rho}{\rho_0},\ -\alpha$
$\phi = \dfrac{p}{\rho_0},\ \Phi$	$\gamma = -1,\ g\rho_{\text{FW}}$

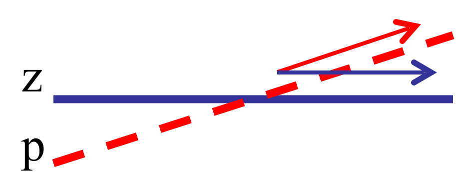
Comparing models in different coordinates

- Comparing fields in z and p requires interpolation
  - interpolation error can be bigger than signal!



- Only depth integrated fields can be directly compared:
  - Sea surface elevation (SSH)
  - Bottom pressure (weight of water column)

- Representing processes along z and p surfaces is different
  - differences between models in z and p may be due such inconsistencies and *not* due to non-Boussinesq effects
  - position of free surface variable!!!



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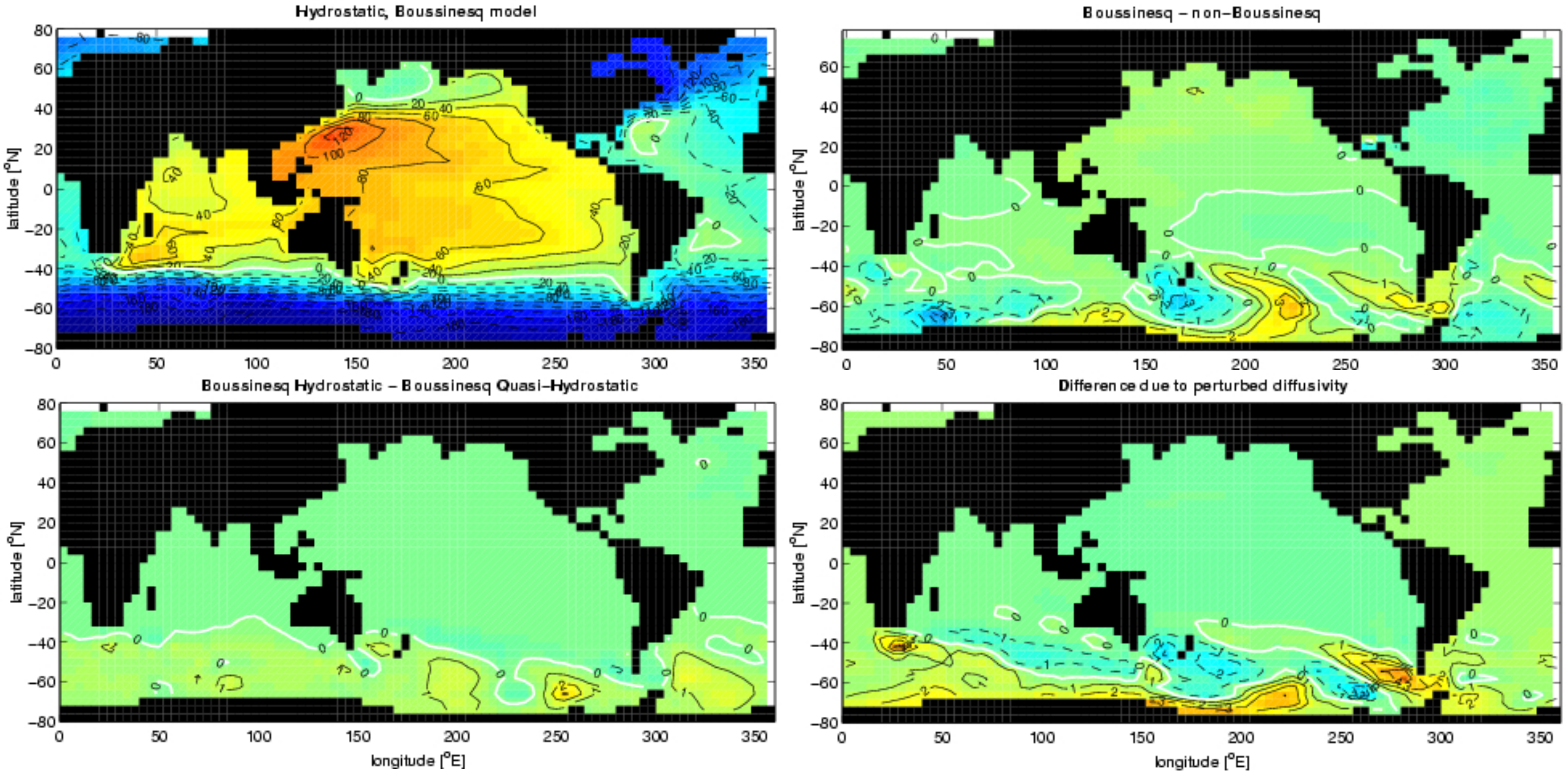
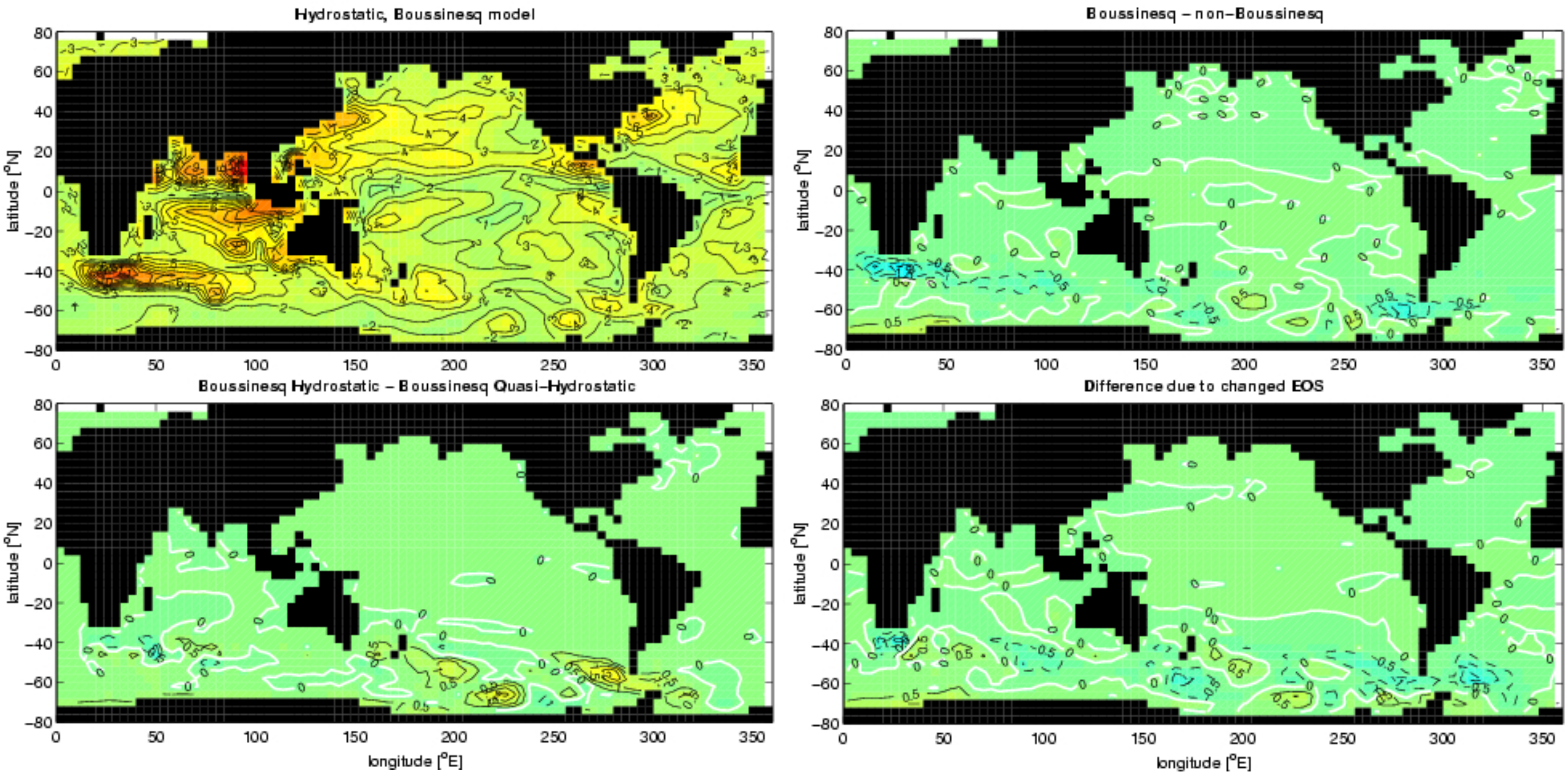
Changes to mean SSH (cm)



- 4˚ model, between 80˚N/S, 15 levels, no sea ice, simple convective adjustment, no eddy parameterisation scheme, nonlinear free surface (Losch et al, 2004)

Changes to SSH variability (cm)



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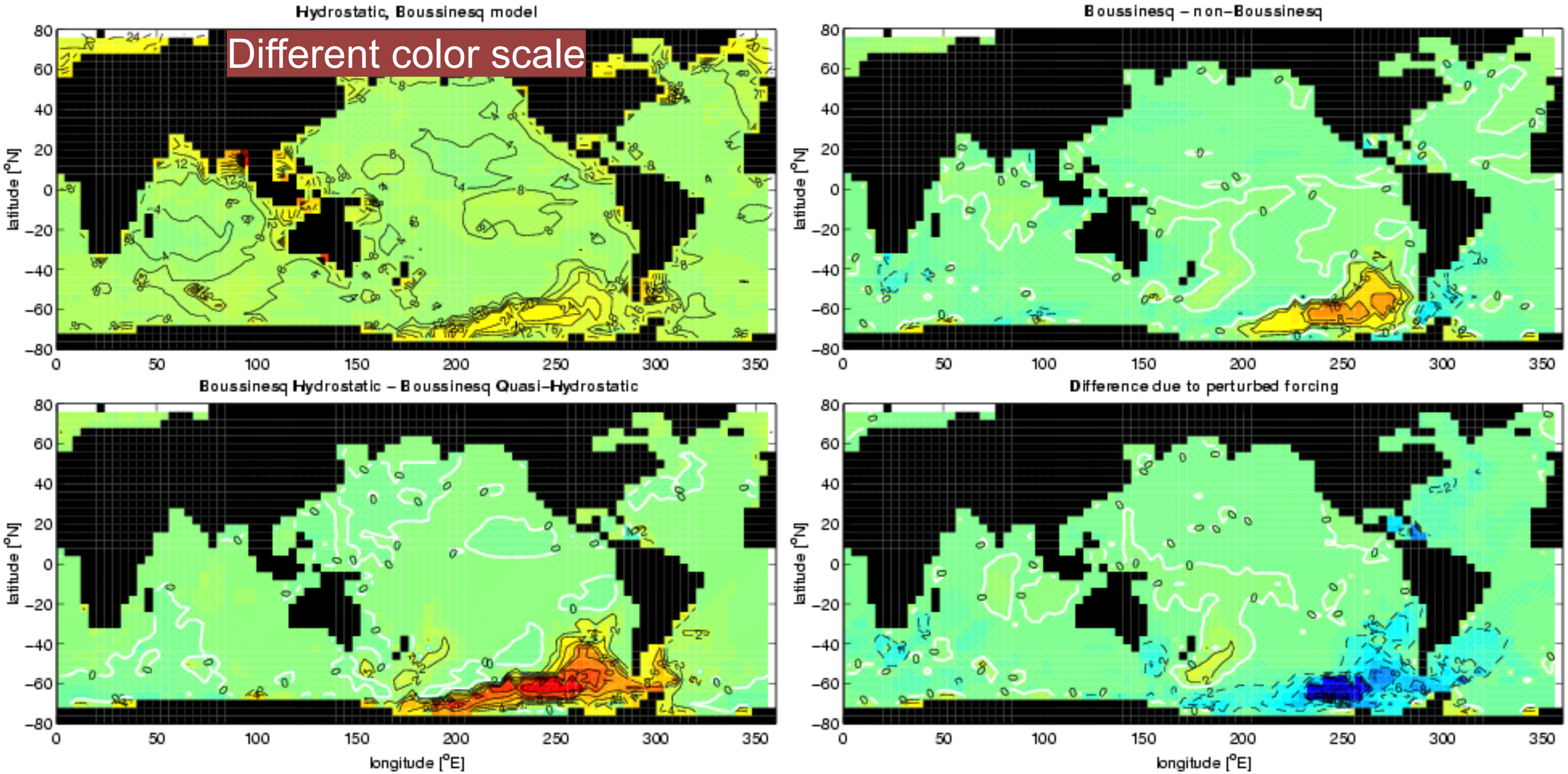
Variability in Bottom Pressure (cm)



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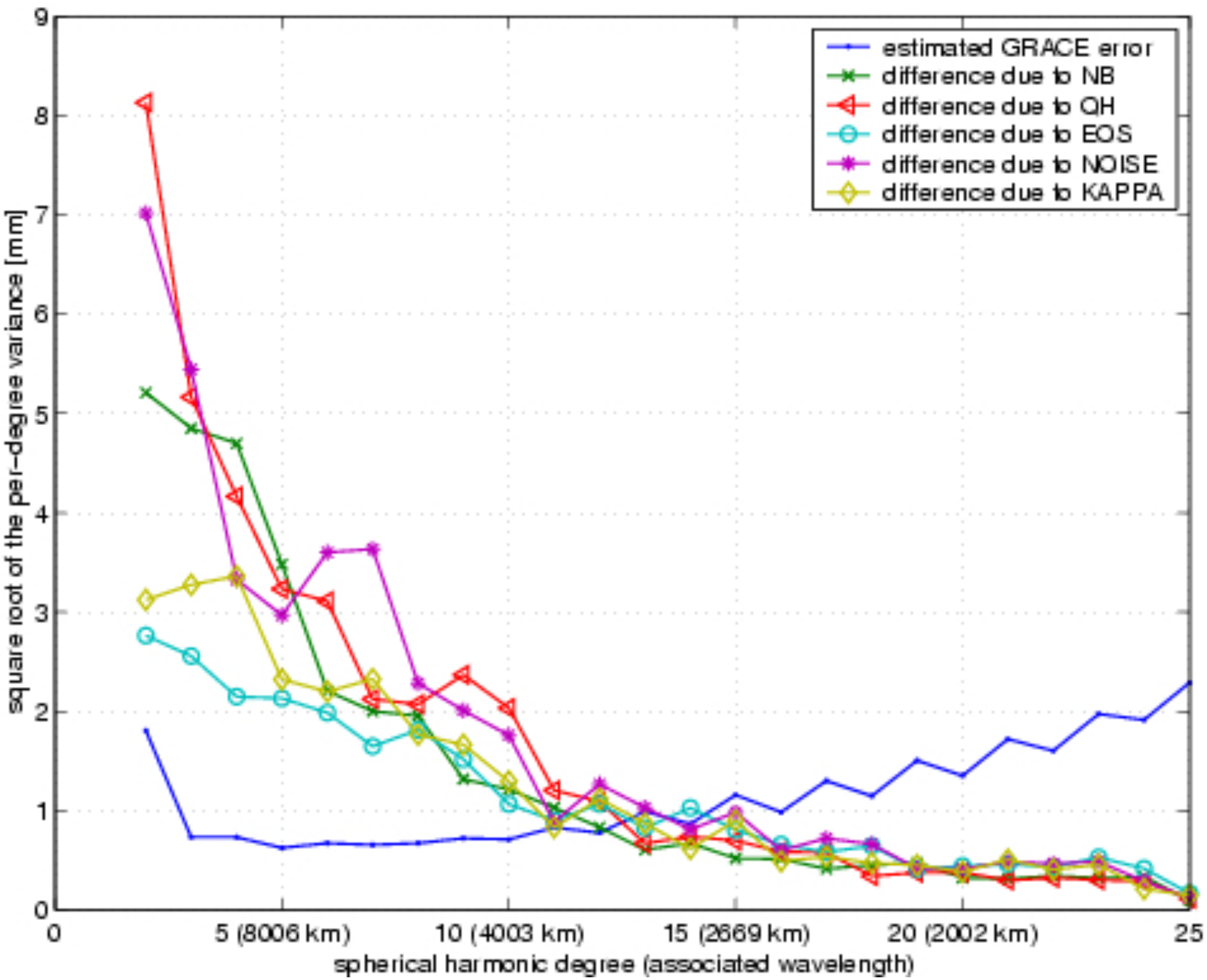
How important are these effects?



Losch et al. (2004), 4deg grid

- < 2000km
  - error in GRACE data larger than effects
- > 2000km
  - effects in model no bigger than due to numerical noise!
- coarse resolution!!!!
- *Story may change at higher resolution*

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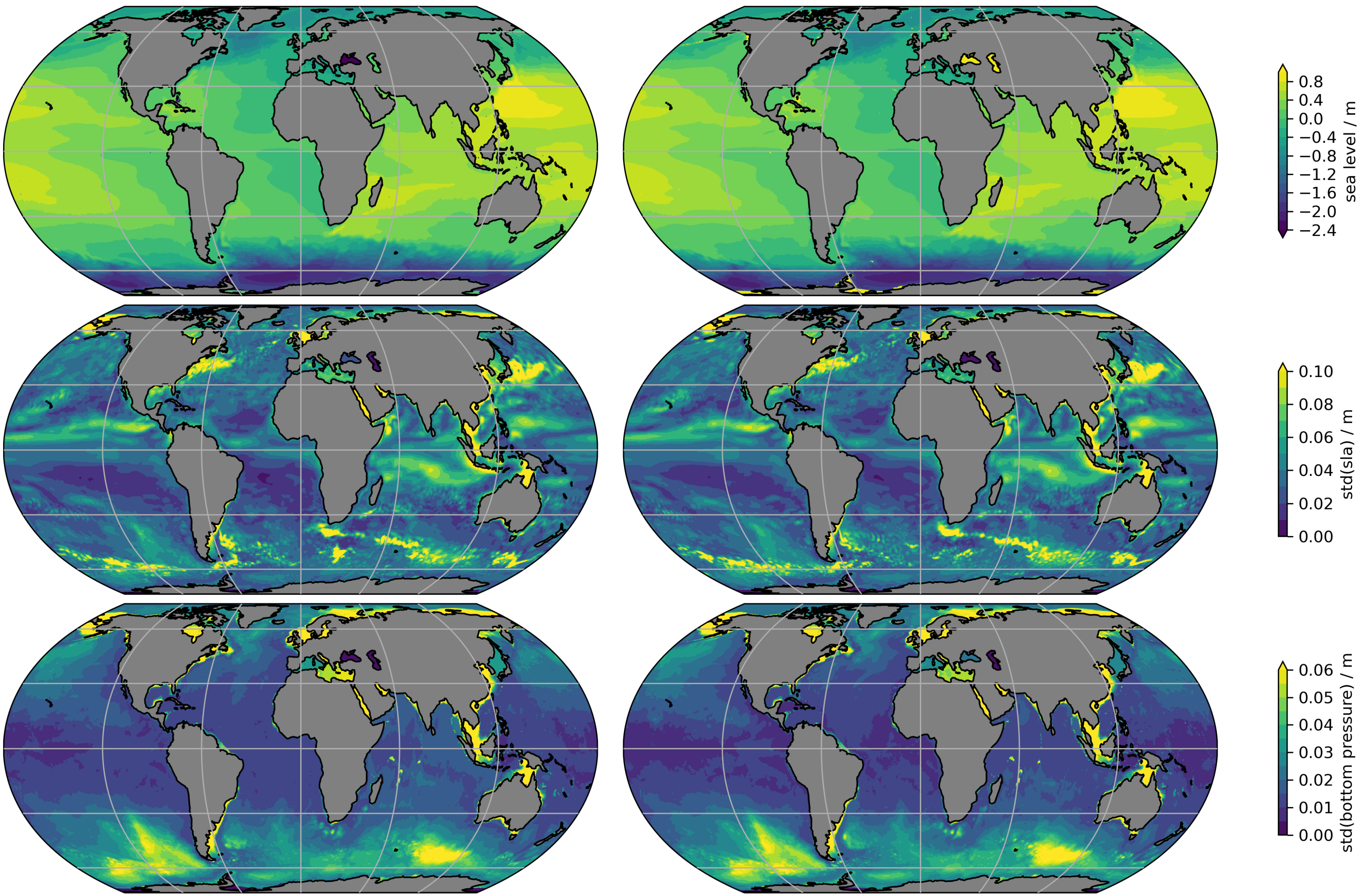
for a coarse resolution general circulation model

- Boussinesq and hydrostatic approximations have similar effects on the circulation
- effects due to numerical truncation error and unclear parameterisations are of similar magnitude
- even coarse models are sensitive to small changes in dynamics and forcing!!!!!

Update: increase complexity and resolution

- LLC270 (1/3°), 50 levels, some “eddies”
- sea ice model (levitating)
- Gent-McWilliams and Redi-scheme ($\kappa_{GM} =$ 80 ms^{-2})
- Vertical mixing scheme: TKE (Gaspar et al, 1990) + IDEMIX (Olbers and Eden, 2013, Eden and Olbers, 2014)

z vs p-coordinates (year 62)



z-coordinates (Boussinesq) p-coordinates (non-Boussinesq)

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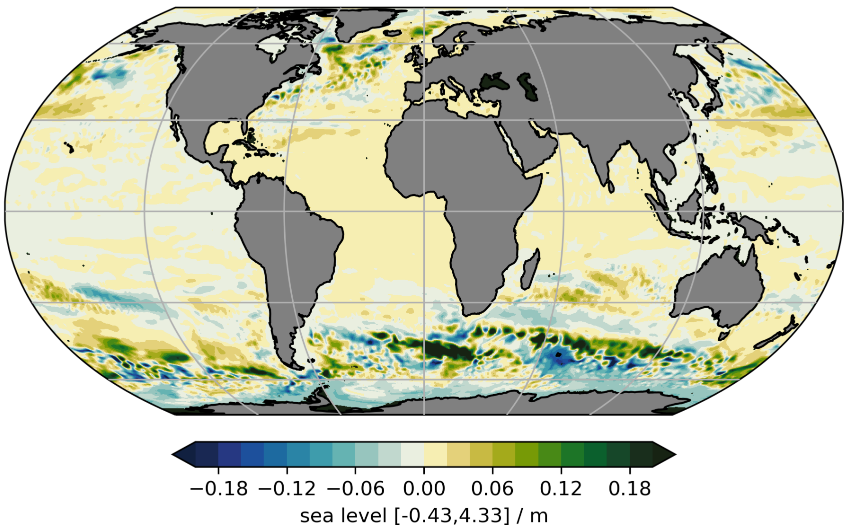
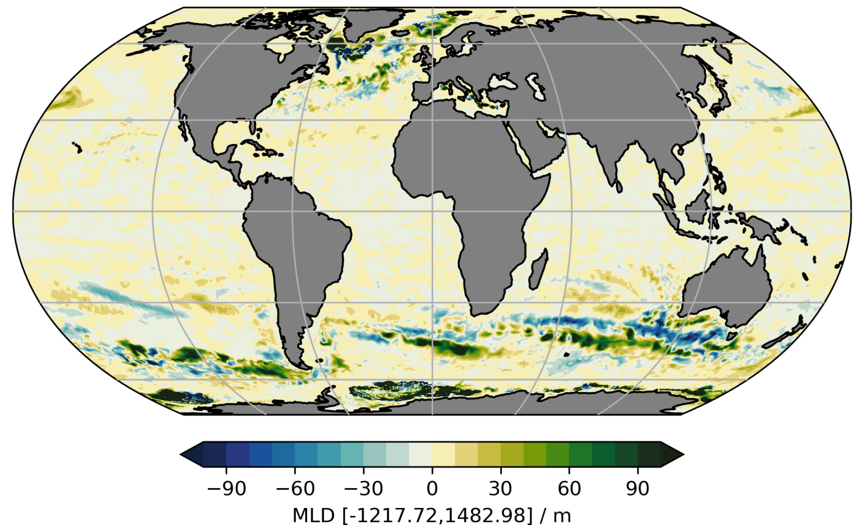
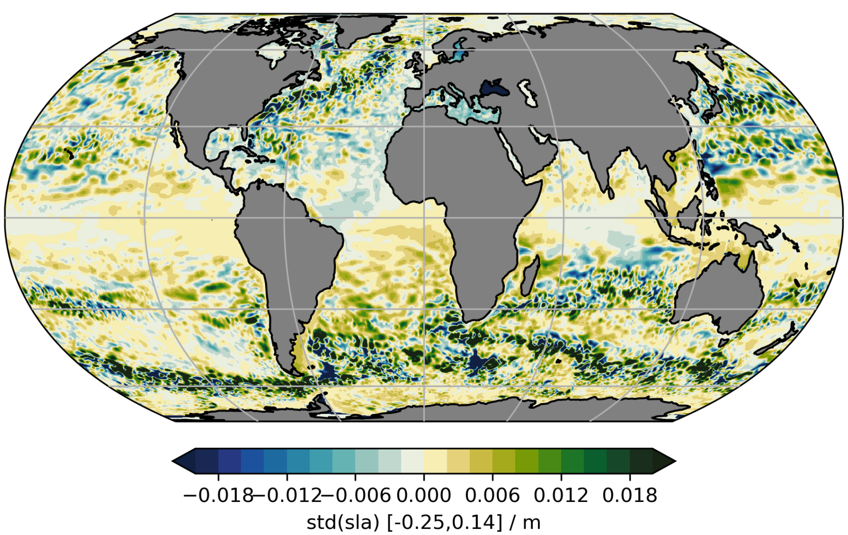
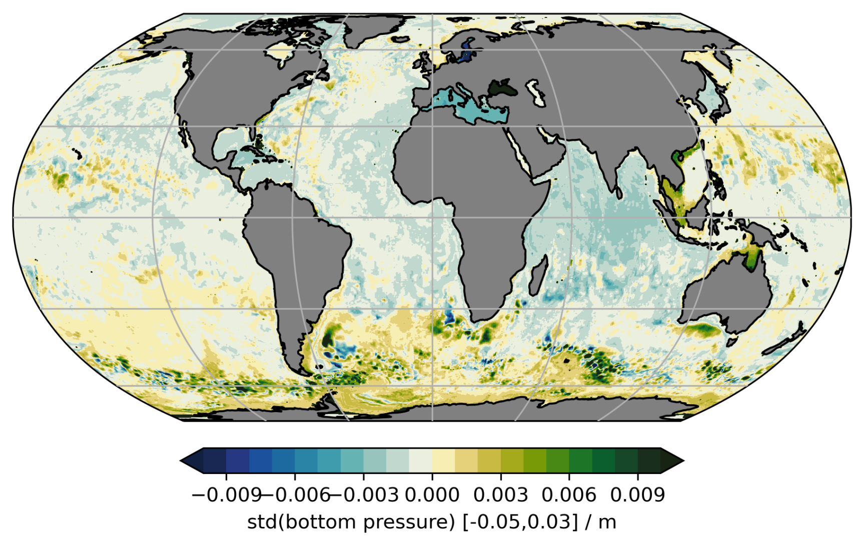
differences small but systematic?

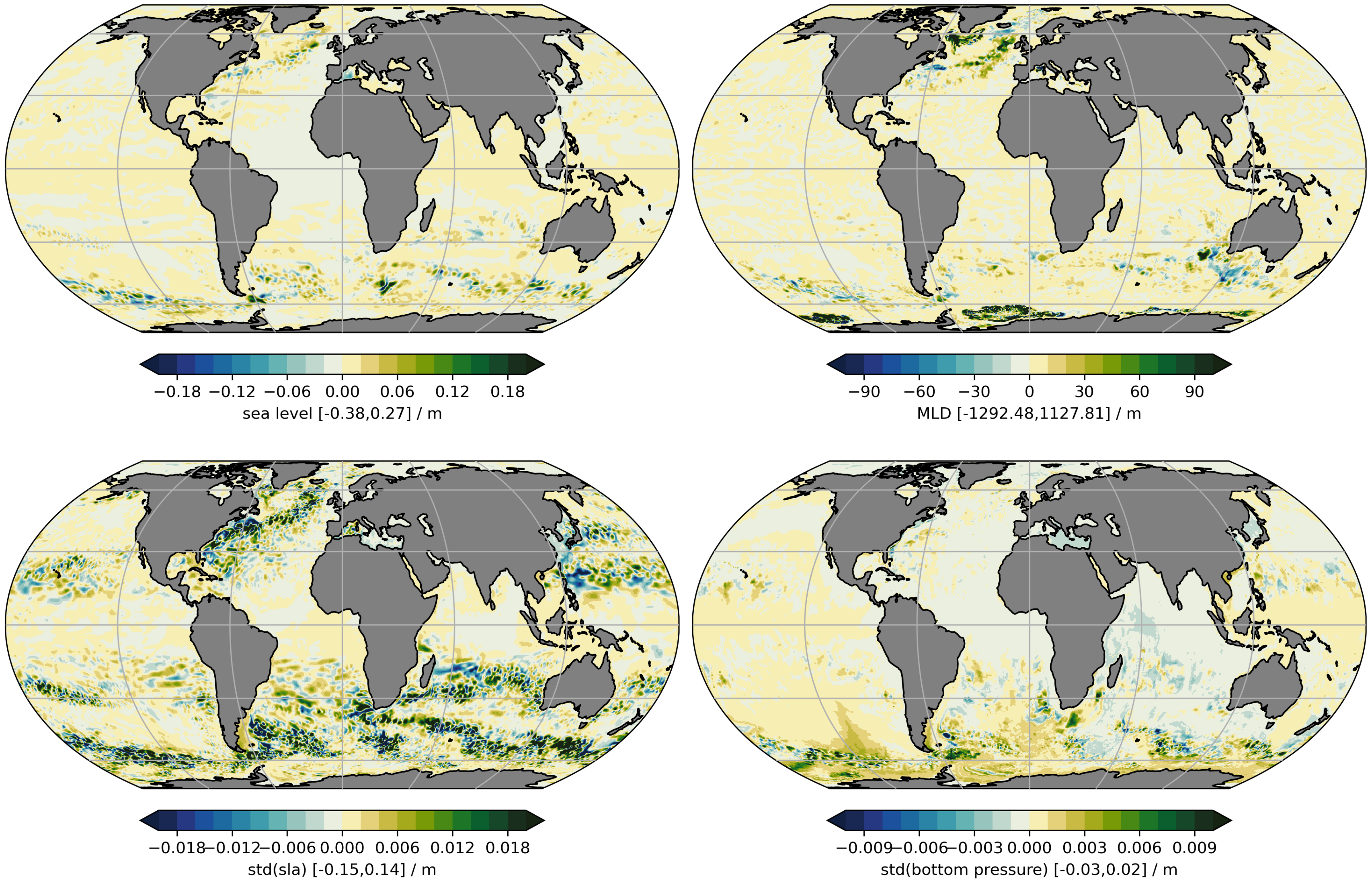




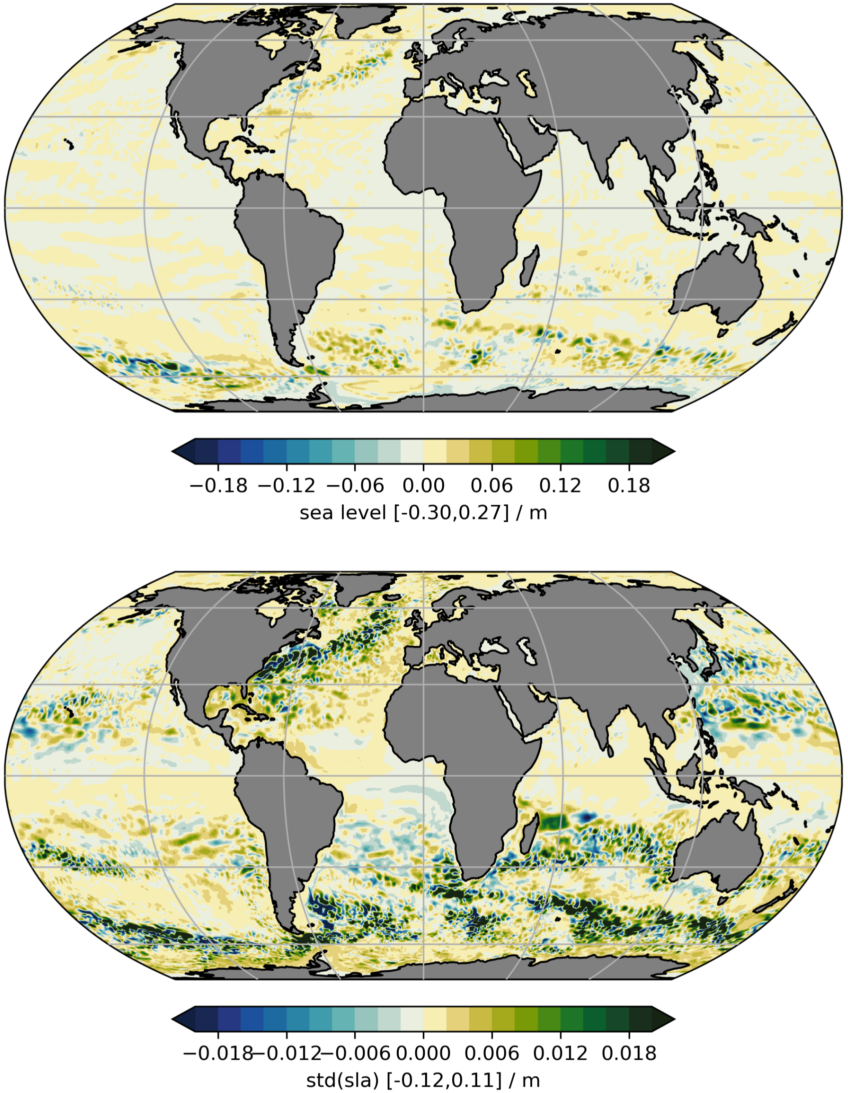




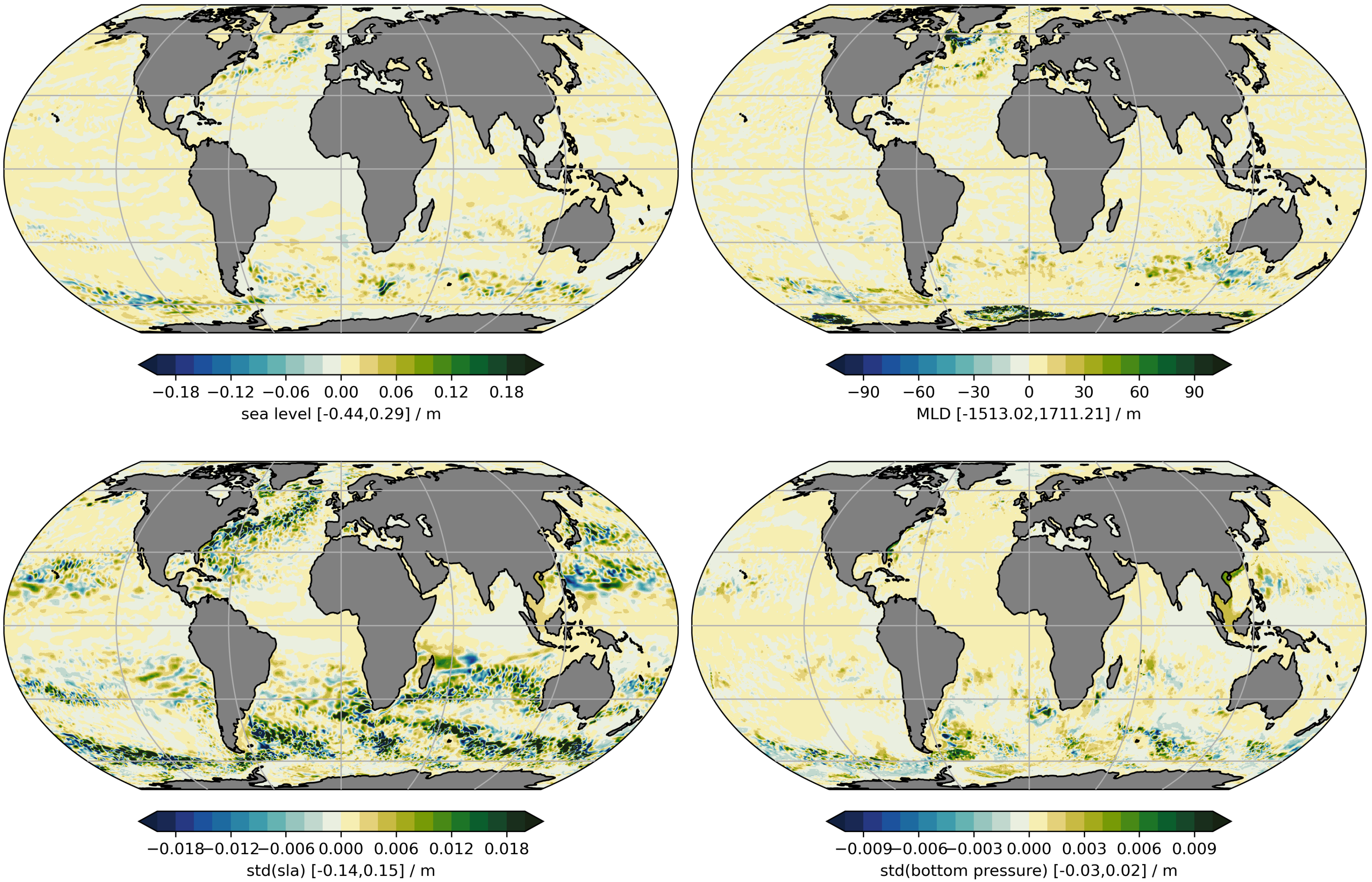
Difference due to EOS



Difference due to quasi-hydrostatic approx.

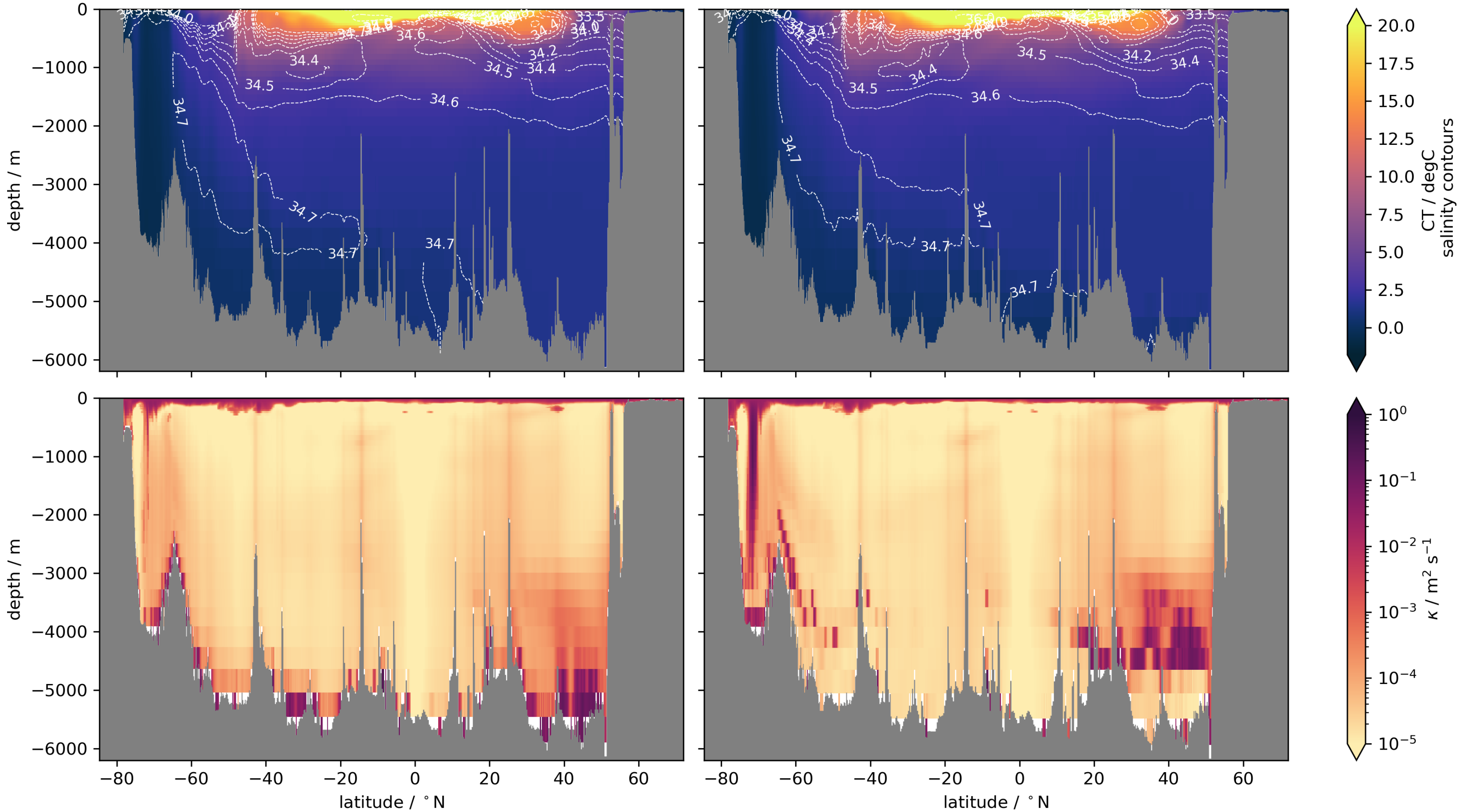


Difference due to model numerics



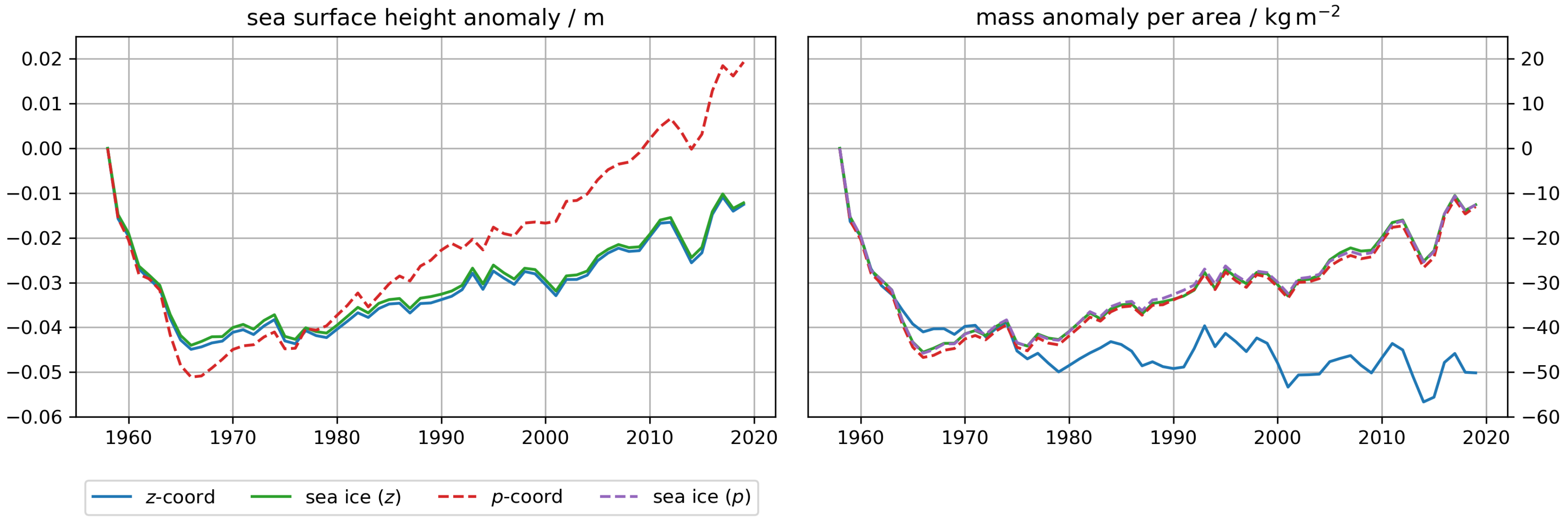
z vs p-coordinates (year 62)

- with IDEMIX!!!
- section through the Pacific Ocean

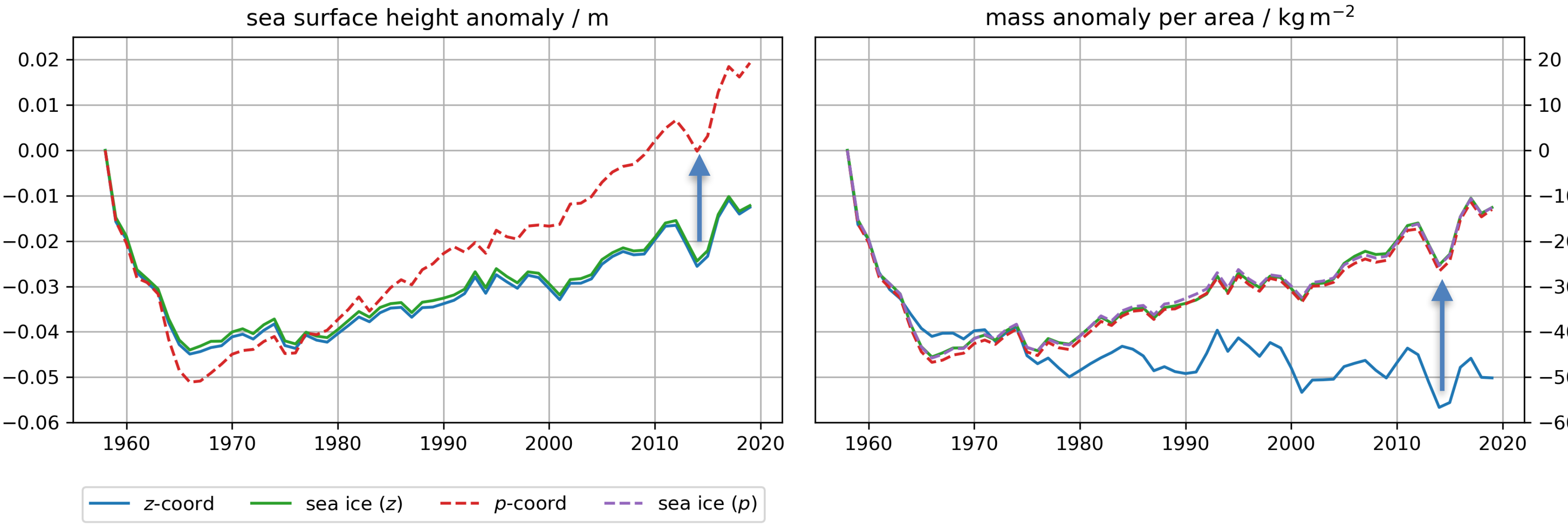


z-coordinates (Boussinesq) p-coordinates (non-Boussinesq)

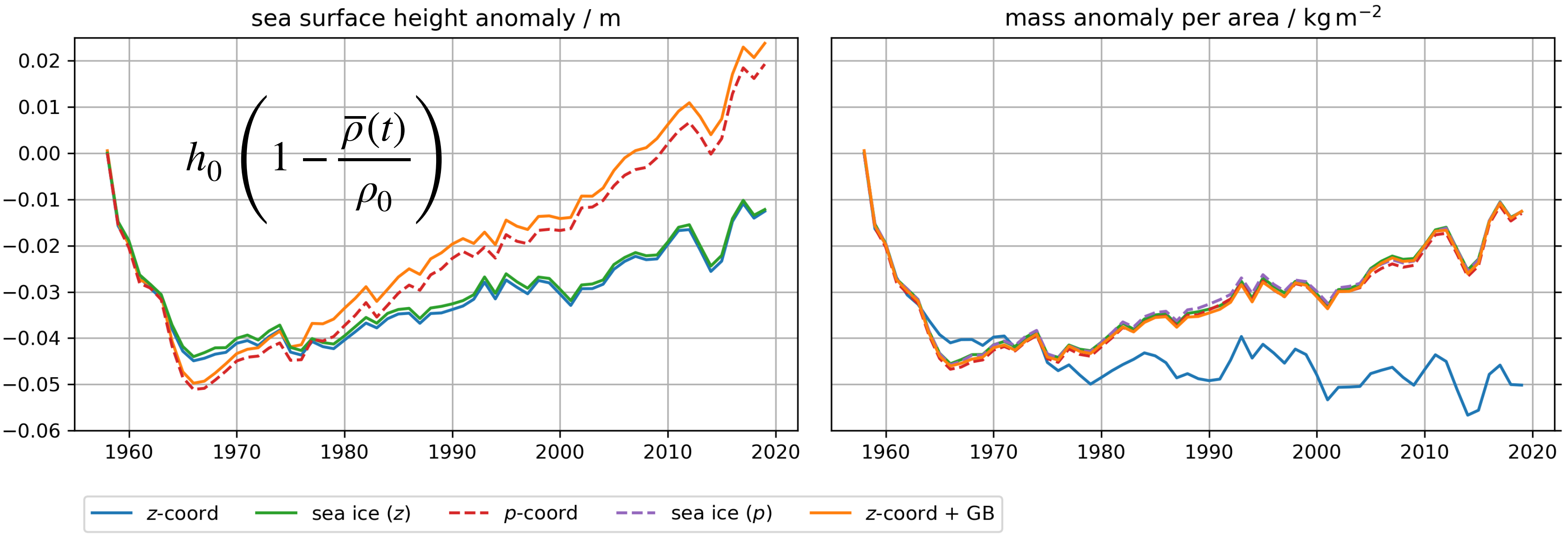
Mean volume and mass



Mean volume and mass

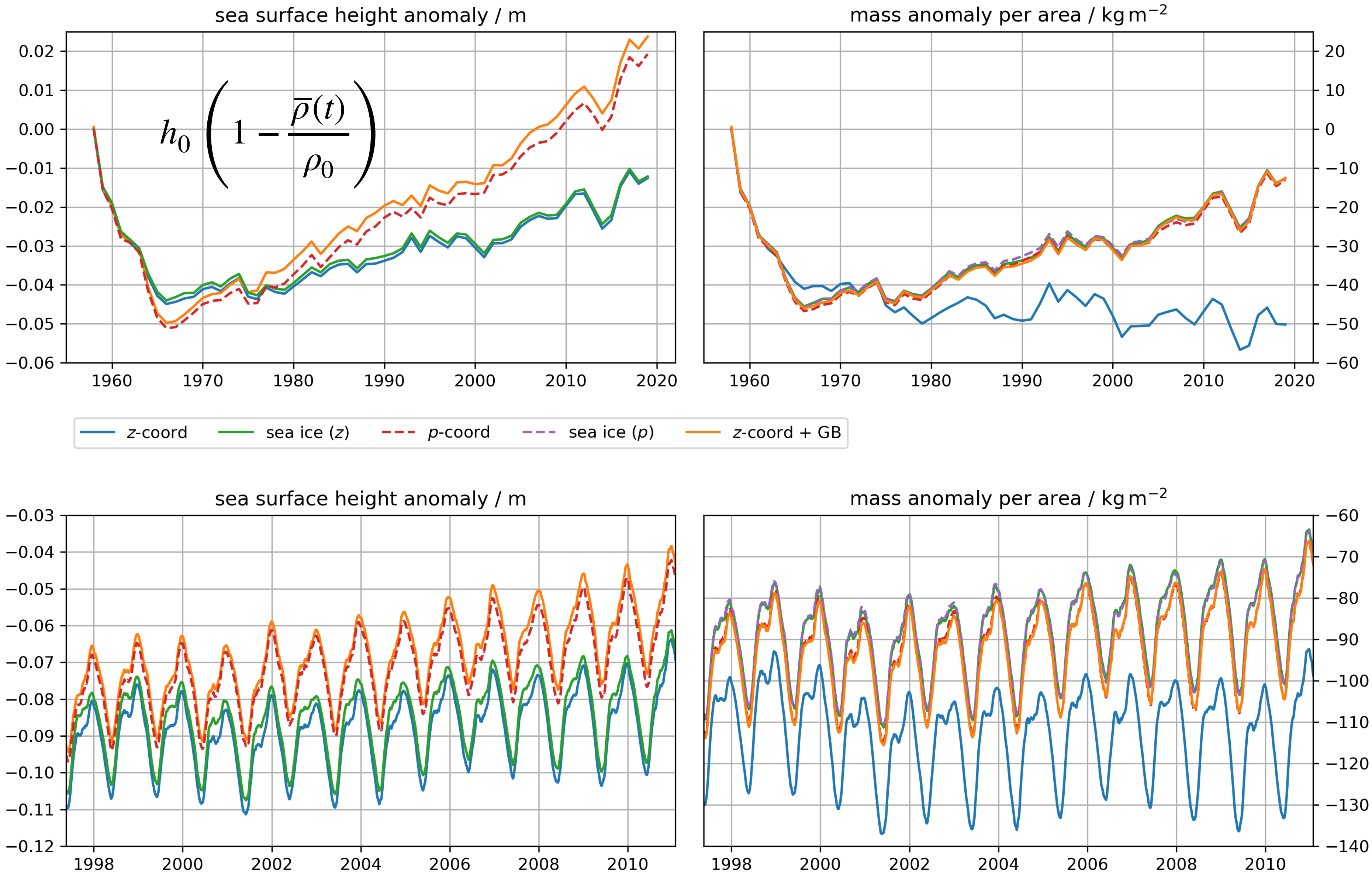


Mean volume and mass

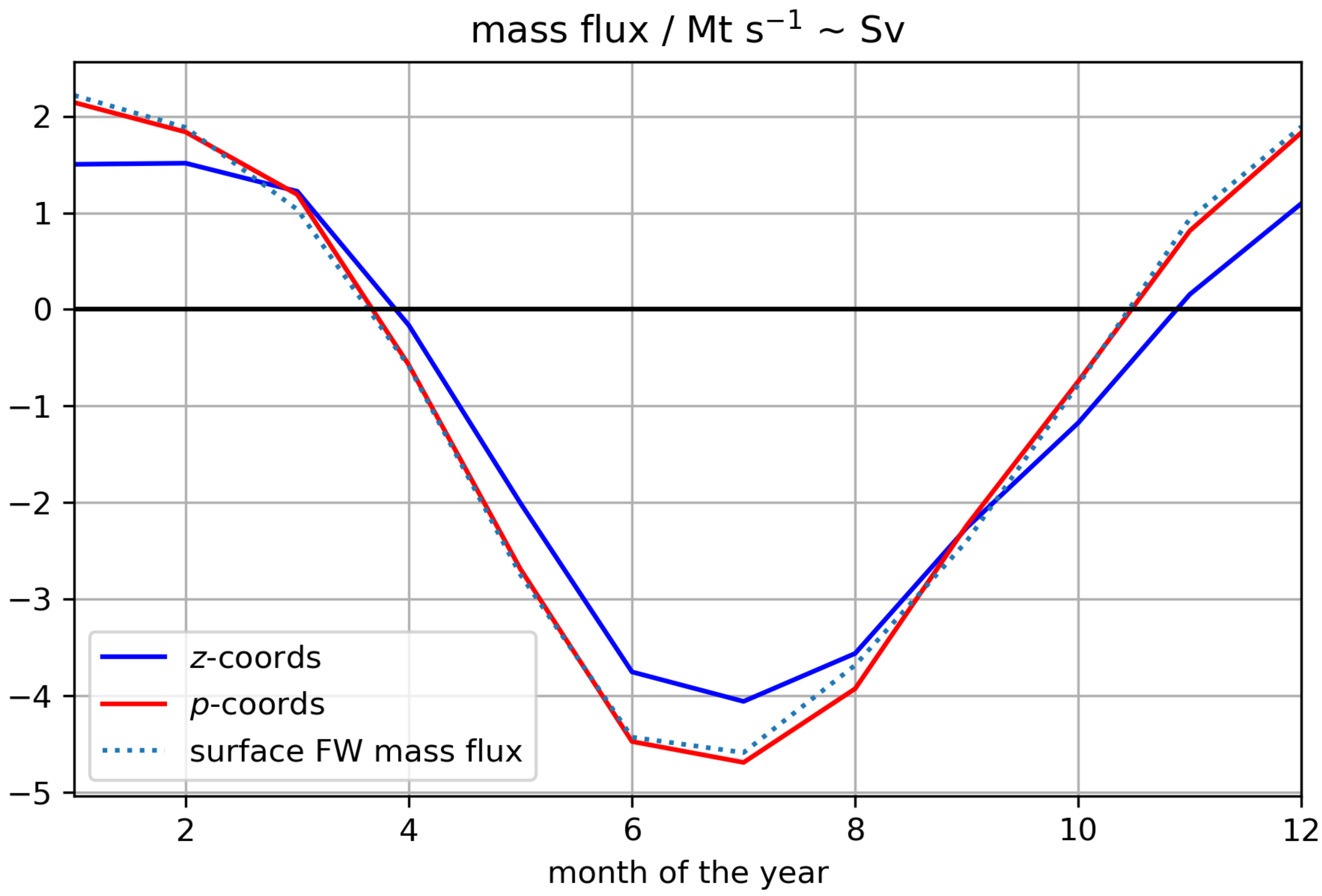


$$h_0\left(1-\frac{\overline{\rho}(t)}{\rho_0}\right)$$

Mean volume and mass



Seasonal equatorial transport



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at higher resolution

- Greatbatch (1994) correction is still great
- differences between non-Boussinesq and Boussinesq model may be larger at higher resolution, maybe even systematic, but still at the level of other uncertainties (here, EOS, numerics, quasi-hydrostatic approximation) (But more careful comparison required: initial conditions, …)
- Order (10%) of cross-equator mass transport not resolved in Boussinesq model
- Replacing pressure by mass coordinates (gp) conveniently solves forcing issue by atmospheric pressure (to be done)