

Diagnosing numerical mixing by Discrete Variance Decay in an unstructured OGCM

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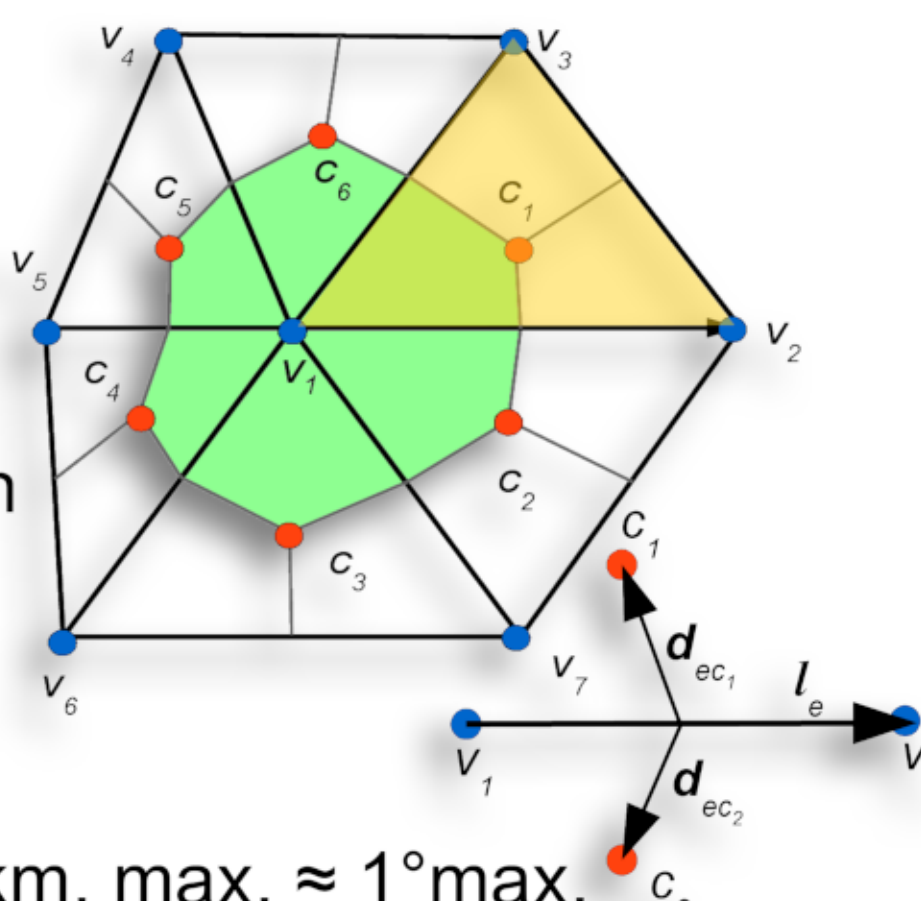
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- FESOM2 is a multi-resolution ocean general circulation model developed by the Alfred Wegener Institute.
- It uses finite-volume methods on unstructured computational grids, allowing flexible representation of complex geometries and fine resolution in regions of interest.
- By adopting a prism-based mesh and improved numerical schemes, FESOM2 achieves computational efficiency comparable to structured mesh models, making it highly competitive.
- Last Release: FESOM2.6
- Setup: core2 mesh (min. $\approx 20\text{km}$, max. $\approx 1^\circ$ max. resolution.), 48 vertical levels, ALE z^* vertical coordinates, TKE mixing scheme., using Redi-GM
- Github: <https://github.com/FESOM/fesom2>



Discrete Variance Decay (DVD):

- What is DVD?** It is a method to diagnose spurious numerical mixing in ocean models by analyzing the loss of tracer variance during advection and diffusion steps. DVD quantifies the contribution of discretization errors to artificial mixing while distinguishing it from physically parameterized processes. The rates are used to evaluate how numerical schemes redistribute variance without affecting overall tracer conservation. This diagnostic tool helps identify and reduce sources of numerical mixing thereby improving model reliability
- DVD has been implemented in FESOM2 as an online diagnostic tool using two different methods:

$$\partial_t T + \nabla \cdot (\mathbf{v}T - \kappa \nabla T) = 0$$

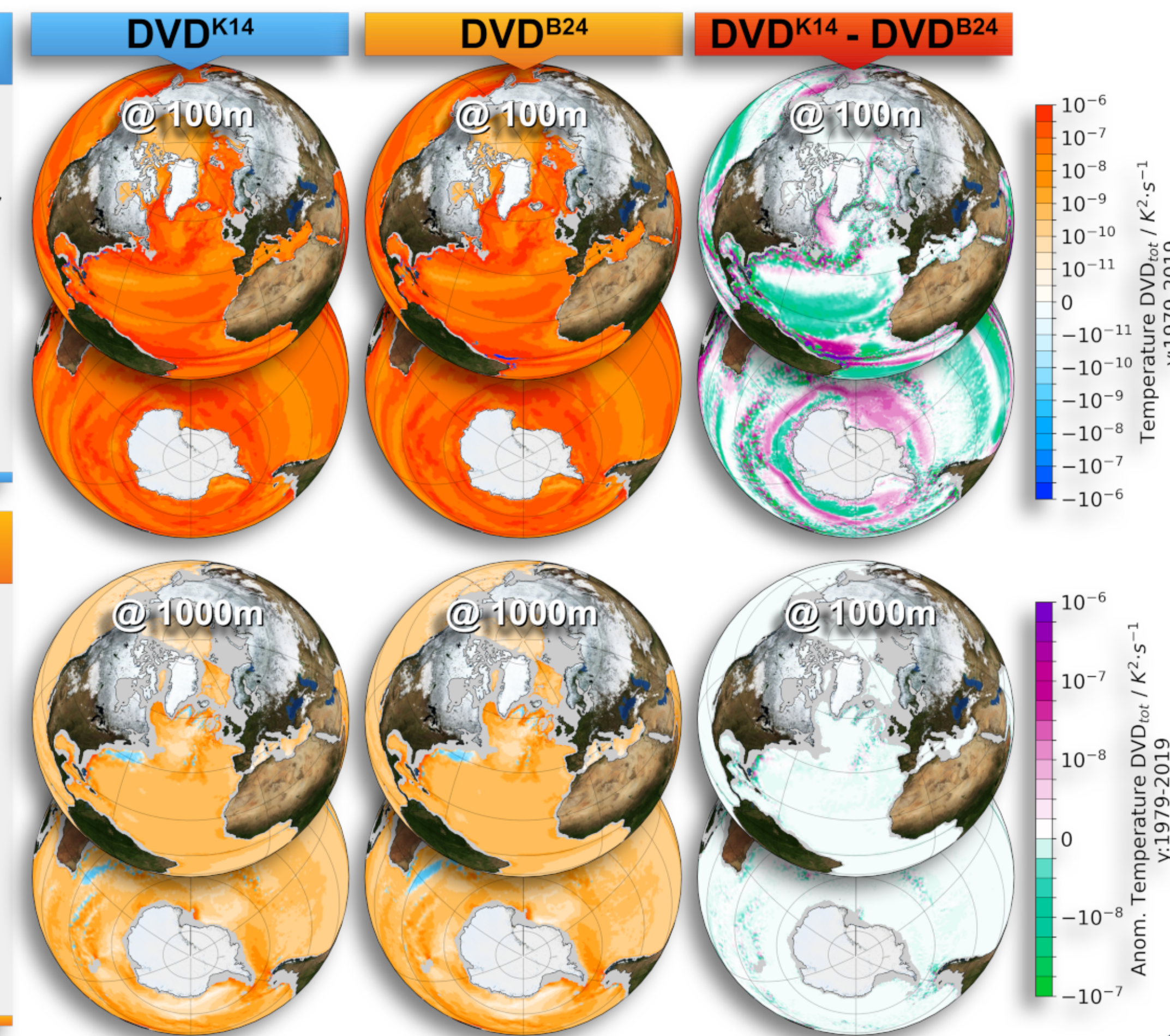
$$\partial_t T^2 + \nabla \cdot (\mathbf{v}T^2 - 2T\kappa \nabla T) = -2\nabla T \cdot \kappa \nabla T = -\chi$$

(1) Klingbeil et al. 2014 (K14):

- Diagnose the DVD rate with respect to the cell centers by a reasonable choice of the fluxes of 2nd moments based through the cell faces and their decomposition and recombination into cell related subvolumes.
- FESOM2 doesn't use directional operator splitting when solving tracer equation, therefore with K14 can only compute total DVD rate

(2) Banerjee et al. 2024 (B24):

- Tries to find direct expression for the local DVD rates of the different subprocesses only by using the discretised 1st moment tracer fluxes and changing the viewpoint from cell centers to the edge faces.
- Has the weakpoint that the local DVD rates can still contain contributions from the first moment flux divergence, which requires a coarse-graining in space or time to eliminate this effect in the local DVD rates.



DVD component splitting with the method of B24:

- In FESOM2 and other OGCMs like MOM6, MPAS, and POP, tracer operator splitting is avoided due to performance constraints, restricting the K14 method to calculating only the total DVD rate for all tracer operations combined. In contrast, the B24 method enables directional splitting of DVD rates for individual tracer operations, even without employing tracer operator splitting, offering greater flexibility.

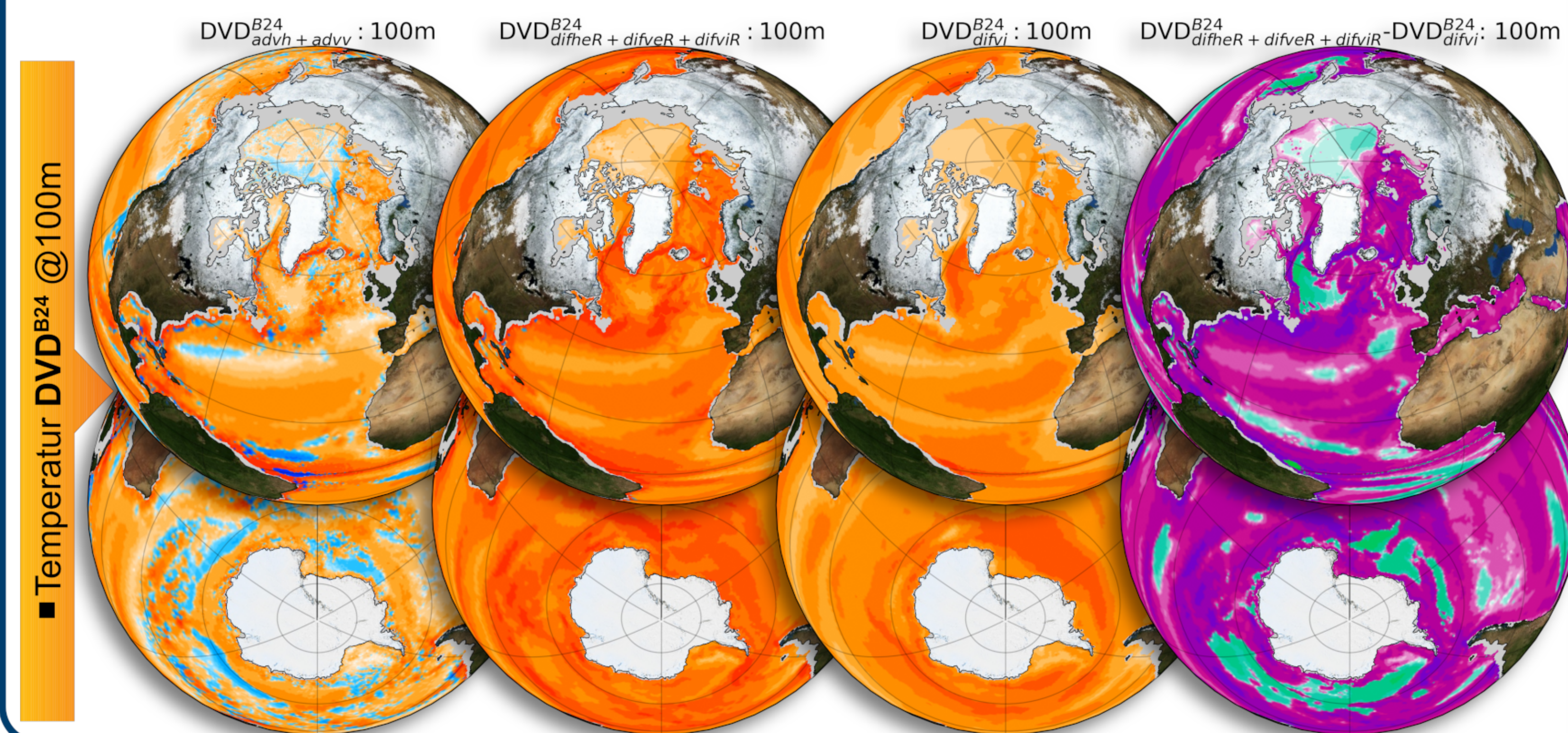
Numerical mixing **Redi mixing** **"Physical" mixing** **Redi-Phys.**

artificial mixing caused by truncation, discretization errors in numerical schemes on grid resolution

iso-neutral mixing along isopycnals is a "combined" numerical and physical mixing (unconstrained magnitude)

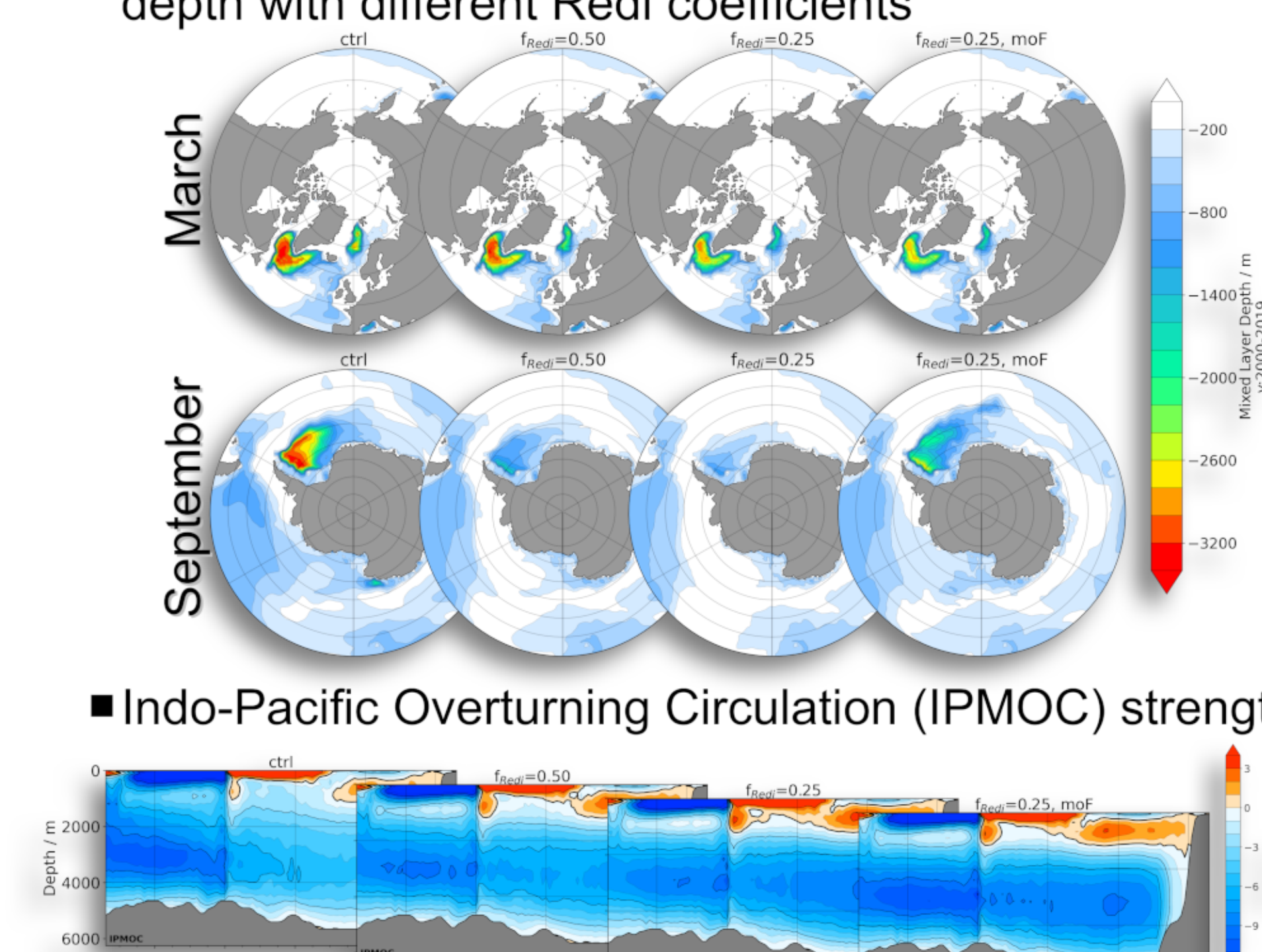
parameterization based on physical processes like, shear, buoyancy, turbulence and subgrid-scale processes

Ideally "physics" controlled parameterized mixing should exceed numerical mixing and Redi mixing



Overestimation of Redi:

- DVD analysis shows that in FESOM2 the Redi mixing in the ocean interior is in many places stronger than the imposed "physical" mixing through the chosen vertical mixing scheme. This might mask effects of parameterisations like IDEMIX
- In FESOM2 the coefficient of GM and Redi are synchronized using a relatively large value of 2000-3000
- We made some test runs with a reduced Redi coefficient, evaluating the deep water formation in the Labrador and Weddell Sea
- Maximum NH March and SH September mixed layer depth with different Redi coefficients
- Indo-Pacific Overturning Circulation (IPMOC) strength



Outlook:

- Ocean model dynamics are driven by density, making it essential to extend DVD analysis for diagnosing spurious numerical mixing in density fields.
- Apply DVD analysis to compare numerical mixing across ocean models with similar complexity and resolution.
- Utilize DVD analysis to assess the impact of varying mesh resolutions on spurious numerical mixing in FESOM2.
- Leverage DVD analysis to evaluate improvements in numerical mixing achieved through higher-order advection schemes in FESOM2.



- TriPyView version 3.0 (<https://github.com/FESOM/tripyview>) is a triangular plotting and diagnostic toolbox for FESOM2, optimized for small to medium meshes.
- It supports multi-panel layouts, command-line processing via Papermill, and features updated xarray-based data handling and Cartopy-integrated plotting. With a Dask client, data reading and processing are parallelized.
- Provides diagnostics to compute z-level and density-based Meridional Overturning Streamfunctions, along with various flux-related variables, on unstructured FESOM2 meshes.